VANDERBILT UNIVERSITY

MATH 4110 – PARTIAL DIFFERENTIAL EQUATIONS

Some notation and terminology

We will denote

$$\mathbb{R}^n = \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{n \text{ times}}.$$

Thus, an element $x \in \mathbb{R}^n$ is an ordered n-tuple

$$x = (x^1, x^2, \dots, x^n).$$

Notice that we denote the components of x with superscripts (although sometimes subscripts will also be used, i.e., $x = (x_1, x_2, \dots, x_n)$). We think of elements of $\mathbb{R}^n$ as vectors, so that the usual vector operations (addition, multiplication by scalars, etc.) are defined. For instance

$$(x^1, x^2, \dots, x^n) + (y^1, y^2, \dots, y^n) = (x^1 + y^1, x^2 + y^2, \dots, x^n + y^n).$$

We will not employ any special notation (such as $\vec{x}$ or $\mathbf{x}$) to denote vectors.

When $n = 2$ or $n = 3$, we sometimes use (x, y) and (x, y, z) to denote (x^1, x^2) and (x^1, x^2, x^3), respectively. The value of n is sometimes called the dimension of the space ($\mathbb{R}^2$ is two-dimensional, $\mathbb{R}^3$ is three-dimensional).

In many scenarios, we will take $x \in \mathbb{R}^n$ to be an independent variable. Hence, calculus operations such differentiation, divergence, etc., are defined, e.g.,

$$\operatorname{div}(x) = \frac{\partial x^1}{\partial x^1} + \cdots + \frac{\partial x^n}{\partial x^n} = n.$$

The components x^i of x, $i = 1, \dots, n$, are called coordinates and for them calculus operations also apply, e.g.,

$$\frac{\partial x^1}{\partial x^2} = 0, \ \frac{\partial x^3}{\partial x^3} = 1.$$