VANDERBILT UNIVERSITY

MATH 4110 – PARTIAL DIFFERENTIAL EQUATIONS

HW 4

Recall that in class we solved the the wave equation with Dirichlet boundary conditions by supposing that $u(x,t) = X(x)T(t)$. This is called the method of separation of variables. Below, you are asked to employ this method for solving other problems.

Question 1. Use separation of variables to solve the following initial-boundary value problem for the wave equation (the only difference from what was done in class is the boundary condition):

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= 0 && \text{in } (0,L)\times(0,\infty),\\ u(x,0) &= f(x) && 0 \le x \le L,\\ u_t(x,0) &= g(x) && 0 \le x \le L,\\ u_x(0,t) &= 0 && t \ge 0,\\ u_x(L,t) &= 0 && t \ge 0. \end{aligned}$$

Question 2. Show that the solution you found in problem 1 can be written as a superposition of a forward and a backward wave.

Question 3. Solve problem 1 with $c = 1$, $L = \pi$, $f(x) = \sin^3 x$, and $g(x) = \sin(2x)$.

Question 4. Use separation of variables to solve the following initial-boundary value problem for the heat equation:

$$\begin{aligned} u_t - k u_{xx} &= 0 && \text{in } (0,L)\times(0,\infty),\\ u(x,0) &= f(x) && 0 \le x \le L,\\ u(0,t) &= 0 && t \ge 0,\\ u(L,t) &= 0 && t \ge 0. \end{aligned}$$

Interpret your result.

Question 5. Solve problem 4 with $k = 17$, $L = \pi$, and

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{\pi}{2},\\ 2, & \frac{\pi}{2} < x \le \pi. \end{cases}$$

Discuss the convergence of the solution you found.