STUDY QUESTIONS FOR THE FIRST TEST

MATH 3120

Unless stated otherwise, the notation below is as in class. You can assume that all functions are C^∞ unless stated otherwise However, if you are asked to state a specific theorem, state it with the regularity as proved in class/homework.

Question 1. (a) Show that solutions to the $1d$ wave equation can be written as a sum of a right-moving and a left-moving wave.

(b) State D'Alembert's formula.

(c) Prove D'Alembert's formula.

Question 2. (a) Give the definition of classical and generalized solutions to the $1d$ wave equation.

(b) Prove that singularities in solution to the $1d$ wave equation propagate along the characteristics.

Question 3. (a) State the mean value theorem for harmonic functions.

(b) Prove the mean value theorem for harmonic functions.

(c) State the converse of the mean value theorem for harmonic functions.

(d) Prove the converse of the mean value theorem for harmonic functions.

Question 4. (a) State the maximum principle for harmonic functions.

(b) Prove the maximum principle for harmonic functions.

(c) State and prove the minimum principle for harmonic functions.

Question 5. (a) State the finite propagation speed theorem for solutions of the wave equation.

(b) State and prove conservation of energy for solutions of the wave equation with compactly supported data.

Question 6. Show how to solve the inhomogeneous wave equation using Duhamel's principle.

Question 7. Show that there exists a constant $C > 0$ such that for any solution u to the $3d$ wave equation it holds that

$$|u(t,x)| \leq \frac{C}{t} \int_{\mathbb{R}^3} (|D^2 u_0(y)| + |Du_0(y)| + |u_0(y)| + |Du_1(y)| + |u_1(y)|)\, dy$$

for $t \geq 1$.