HOMEWORK 7

MATH 3120

Unless stated otherwise, the notation below is as in class. You can assume that all functions are C^∞ unless explicitly assumed otherwise.

Question 1. Consider continuous dependence on the data for the wave equation in $3d$, where smallness on the data part is measured with respect to the norm

$$\|f\|_2 := \int_{\mathbb{R}^3} (|D^2 f(y)| + |Df(y)| + |f(y)|)\, dy.$$

Give a precise formulation of the continuous dependence on the data and prove your statement.

Hint: Use the estimate of Question 3 in HW 6 as a basis for your statement, and give a similar proof (now you have to also account for $t < 1$).

Question 2. In the equations below, identify the functions $a(t,x,u)$, $b(t,x,u)$, and $c(t,x,u)$.

(a) $(1+t^2)\partial_t u + 3\partial_x u + u^2 = 0$.

(b) $\sin(x)e^t u_t + |u|^3 u_x = 0$.

Question 3. Solve the problem below using the method of characteristics and give a description of the (projected) characteristics.

$$x\partial_t u - t\partial_x u - u = 0,$$
$$u(0,x) = h(x).$$

Question 4. Does the transversality condition hold for the problem of question 3? What can you say about uniqueness and how is it related to the solution you found?