VANDERBILT UNIVERSITY
MATH 294 — PARTIAL DIFFERENTIAL EQUATIONS.
HW 4.

Question 1. Prove the following theorems: Riesz Representation Theorem, Closed Graph Theorem, Hahn-Banach Theorem (real and complex), Banach-Steinhaus Theorem/Principle of Uniform Boundedness. When, in your proofs, you use other theorems of functional analysis, state them.

Question 2. Show that the space of bounded linear maps between two Banach spaces is itself a Banach space.

Question 3. Show that weakly convergent sequences on a Banach space are bounded.

Question 4. Show that a bonded sequence in a Hilbert space has a weakly convergent subsequent.

Question 5. Show that the spaces $C^k(\Omega)$, $L^p(\Omega)$, $L^\infty(\Omega)$, endowed with their natural norms, are Banach spaces. Show that $L^2(\Omega)$ is a Hilbert space. Bonus: show that $C^{k,\alpha}(\Omega)$ is a Banach space. Note: as we discussed in class, when we say that, for instance, $C^k(\Omega)$ is a Banach space, we mean k-continuously differentiable functions such that the C^k-norm is finite.

Question 6. Identify the dual space of $L^p(\Omega)$.