VANDERBILT UNIVERSITY
MATH 294 — PARTIAL DIFFERENTIAL EQUATIONS.
HW 2.

Unless stated otherwise, Ω is a bounded domain in $\mathbb{R}^n$.

Question 1. Prove uniqueness of solutions to the Dirichlet problem

$$\begin{cases} \Delta u = f & \text{in } \Omega, \\ u = g & \text{on } \partial\Omega, \end{cases}$$

where f and g are given functions (you may have to specify the spaces these functions belong to).

Question 2. Prove that the following Neumann problem

$$\begin{cases} \Delta u = 1 & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases}$$

has no solution. What can you say about the solvability of

$$\begin{cases} \Delta u = f & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = g & \text{on } \partial\Omega, \end{cases}$$

where f and g are given functions?

Question 3. Let u be a solution of

$$\begin{cases} \Delta u - u = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where f is a given function, sufficiently regular up to the boundary. Show that

$$|u| \leq \max_{\overline{\Omega}} |f|.$$