VANDERBILT UNIVERSITY

MATH 2420 – METHODS OF ORDINARY DIFFERENTIAL EQUATIONS

Examples of section 7.9

Question. Solve

$$\begin{cases} y'' + y = \delta(t-\pi) - \delta(t-2\pi), \\ y(0) = 0,\ y'(0) = 1. \end{cases}$$

Solution. Applying the Laplace transform,

$$\begin{aligned} \mathcal{L}\{y'' + y\} &= \mathcal{L}\{\delta(t-\pi) - \delta(t-2\pi)\} \\ s^2 Y(s) - sy(0) - y'(0) + Y(s) &= e^{-\pi s} - e^{-2\pi s} \\ s^2(Y(s) + 1) &= 1 + e^{-\pi s} - e^{-2\pi s}. \end{aligned}$$

Thus,

$$Y(s) = \frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1} - \frac{e^{-2\pi s}}{s^2+1}.$$

Using

$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a),$$

where $F(s) = \mathcal{L}\{f(t)\}$, we obtain, with $F(s) = \frac{1}{s^2+1}$,

$$y(t) = \sin t + \sin(t-\pi)u(t-\pi) - \sin(t-2\pi)u(t-2\pi).$$

This can be simplified to

$$y(t) = \sin t(1 - u(t-\pi) - u(t-2\pi)).$$