VANDERBILT UNIVERSITY

MATH 2420 – METHODS OF ORDINARY DIFFERENTIAL EQUATIONS

Examples of section 2.3

Question 1. Find a solution to the initial value problem

$$\begin{cases} (50+t)x' + x - 8t = 400, \\ x(0) = 10, \end{cases}$$

where $t \geq 0$.

SOLUTIONS.

Question 1. Since $t \geq 0$, we can divide the equation by $50+t$ as this term is never zero, obtaining

$$\frac{dx}{dt} + \frac{x}{50+t} - \frac{8t}{50+t} = \frac{400}{50+t},$$

or,

$$\frac{dx}{dt} + \frac{x}{50+t} = \frac{400+8t}{50+t} = 8\frac{50+t}{50+t} = 8.$$

The equation

$$\frac{dx}{dt} + \frac{x}{50+t} = 8.$$

is a linear first order equation. As showed in class, the general solution to

$$x' + px = q, \tag{1}$$

is

$$x(t) = \Big(\int q(t) e^{\int p(t)\, dt}\, dt + C \Big) e^{-\int p(t)\, dt}. \tag{2}$$

It is **very important** to notice that (2) can only be applied when the equation is written in the form (1), i.e., with the coefficient multiplying x' being one. That's why we had to first divide the equation by $50+t$.

In our case, using (2), we find:

$$x(t) = \frac{4(t^2 + 100t + 125)}{50+t}.$$