

**MATH 2300-04
PRACTICE TEST 1**

VANDERBILT UNIVERSITY

Directions: This practice test should be used as a study guide, illustrating the concepts that will be emphasized in the first test. This does not mean that the actual test will be restricted to the content of the practice. Try to identify, from the questions below, the concepts and methods that you should master for the test. For each question in the practice test, study the ideas and techniques connected to the problem, even if they are not directly used in your solution.

Take this also as an opportunity to practice how you will write your solutions in the test. For this, write clearly, legibly, and in a logical fashion. Make precise statements (for instance, write an equal sign if two expressions are equal; say that one expression is a consequence of another when this is the case, etc.).

The first test will cover the following sections of the textbook: 12.1, 12.2, 12.3, 12.4, 12.5, 12.6, 13.1, 13.2, and 13.3.

Question 1. let \mathbf{u}, \mathbf{v} be vectors in \mathbb{R}^3 and c, d be scalars. For each expression below, identify whether or not it is well-defined.

(a) $\mathbf{u} \cdot \mathbf{v} + c$.

(b) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$.

(c) $\mathbf{u} \times \mathbf{v} + c$.

(d) $(c\mathbf{u}) \times (d\mathbf{v}) + (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$.

Question 2. Sketch the following surfaces in \mathbb{R}^3 .

(a) $x^2 + 2z^2 = 1$.

(b) $x^2 + y^2 + z^2 = 4$ and $z > \sqrt{x^2 + y^2}$.

Question 3. Consider the planes $x + 2y - z + 3 = 0$ and $x + by - z + b + 1 = 0$. Find the values of b for which both planes intersect orthogonally, and then find the equation of the line determined by the intersection.

Question 4. For each vector-valued function below, find its domain and points of discontinuity (if any).

(a) $\mathbf{r}(t) = (t^2, 1, (t + 1)^{-2})$.

(b) $\mathbf{r}(t) = \mathbf{v}(t) \times \mathbf{u}(t)$, where $\mathbf{v}(t) = e^{-t^2}\mathbf{i} + \cos t\mathbf{j} + t^3\mathbf{k}$, $\mathbf{u}(t) = t\mathbf{i} + \sin(\cos t)\mathbf{j} + (t + 2)^4\mathbf{k}$.

(a) $\mathbf{r}(t) = (\ln t, \frac{1}{t-5}, \sqrt{t-2})$.

Question 5. Let \mathbf{r} and f be a differentiable vector-valued function and a differentiable scalar function, respectively, both defined on an interval (a, b) . Show that

$$(f(t)\mathbf{r}(t))' = f'(t)\mathbf{r}(t) + f(t)\mathbf{r}'(t),$$

$t \in (a, b)$.

Question 6. What is the curvature of an ellipse?

Question 7. Let $\mathbf{r}(t) = (\frac{t^2}{2}, \sin t, \cos t)$. Find the length of the curve traced by $\mathbf{r}(t)$ from $t = 0$ to $t = \frac{\pi}{4}$ in two different ways: (a) by using the formula for the length of a curve, (b) by using the arc length function.