MATH 2300-04
PRACTICE FINAL

VANDERBILT UNIVERSITY

Directions: This practice test should be used as a study guide, illustrating the concepts that will be emphasized in the first test. This does not mean that the actual test will be restricted to the content of the practice. Try to identify, from the questions below, the concepts and methods that you should master for the test. For each question in the practice test, study the ideas and techniques connected to the problem, even if they are not directly used in your solution.

Take this also as an opportunity to practice how you will write your solutions in the test. For this, write clearly, legibly, and in a logical fashion. Make precise statements (for instance, write an equal sign if two expressions are equal; say that one expression is a consequence of another when this is the case, etc.).

The final test will cover the following sections of the textbook: 15.3, 15.5, 15.6, 15.7, 15.8, 15.9, 16.1, 16.2, 16.3, 16.4, 16.5, 16.6, 16.7, 16.8, and 16.9 (15.4 will not be on the test). But in a sense, you should view it as cumulative in that you need the previous material to understand these sections.

Question 1. Let D be the solid bounded by $x = 2$, $y = 2$, $z = 0$, and $x + y - 2z = 2$. Express

$$\iiint\limits_D f(x, y, z)\, dV$$

as an iterated integral.

Question 2. Let S be the paraboloid $z = x^2 + y^2$, $0 \leq z \leq 1$.

(a) Sketch the surface S.

(b) Write S as a parametric surface.

(c) Find an expression for the unit normal vector field to S. Orient it so that the normal points downward.

(d) Evaluate

$$\iint\limits_S \mathbf{F} \cdot d\mathbf{S},$$

where

$$\mathbf{F}(x, y, z) = -x\,\mathbf{i} - y\,\mathbf{j} + z\,\mathbf{k}.$$

Question 3. Using a change of variables, evaluate the integral

$$\iint_D \frac{\sqrt{x^2+16y^2+8xy}}{2\sqrt{x^2+4y^2}}\,dA,$$

where D is the region bounded by $x^2+4y^2=4$, $x^2+4y^2=16$, $y-x-1=0$, and $y-x+2=0$, and $x\geq 0$.

Question 4. Let $\mathbf{F}$ be a vector field $\mathbf{F}=P\,\mathbf{i}+Q\,\mathbf{j}$. Let S_2 be the square with vertices at $(-2,-2)$, $(-2,2)$, $(2,2)$, and $(2,-2)$, and S_3 be the square with vertices at $(-3,-3)$, $(-3,3)$, $(3,3)$, and $(3,-3)$. Let $C_2=\partial S_2$ and $C_3=\partial S_3$, both oriented counter-clockwise. Assume that:

- The functions P and Q have continuous partial derivatives in the region $x^2+y^2\geq 1$.
- $\frac{\partial Q}{\partial x}=\frac{\partial P}{\partial y}$ in the region outside S_2.
- $\int_{C_2}\mathbf{F}\cdot d\mathbf{r}=1$.

(a) Evaluate $\int_{C_3}\mathbf{F}\cdot d\mathbf{r}$.

(b) Is $\mathbf{F}$ a conservative vector field in the region $x^2+y^2\geq 1$?

(c) What can you say about the functions P and Q in the region $x^2+y^2\leq 1$?

Question 5. Use the divergence theorem to evaluate

$$\iint_S \mathbf{F}\cdot d\mathbf{S},$$

where $\mathbf{F}(x,y,z)=3xy^2\,\mathbf{i}+xe^z\,\mathbf{j}+z^3\,\mathbf{k}$, and S is the surface of the solid bounded by the cylinder $y^2+z^2=1$ and the planes $x=-1$ and $x=2$.

Question 6. Carefully review the examples done in class using the fundamental theorem of line integrals, Green's theorem, Stokes' theorem, and the divergence theorem.