

VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 14.3

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Question 1. If f and g are twice differentiable functions of a single variable, show that the function

$$u(x, t) = f(x + at) + g(x - at)$$

is a solution of the wave equation

$$u_{tt} - a^2 u_{xx} = 0,$$

where a is a constant.

Solution 1. Using the chain rule for single variable functions,

$$\frac{\partial}{\partial x} f(x + at) = f'(x + at) \frac{\partial(x + at)}{\partial x} = f'(x + at).$$

Taking another derivative with respect to x ,

$$\frac{\partial^2}{\partial x^2} f(x + at) = \frac{\partial}{\partial x} f'(x + at) = f''(x + at) \frac{\partial(x + at)}{\partial x} = f''(x + at).$$

Similarly

$$\frac{\partial}{\partial t} f(x + at) = f'(x + at) \frac{\partial(x + at)}{\partial t} = af'(x + at),$$

and taking another derivative with respect to t ,

$$\frac{\partial^2}{\partial t^2} f(x + at) = \frac{\partial}{\partial t} (af'(x + at)) = af''(x + at) \frac{\partial(x + at)}{\partial t} = a^2 f''(x + at).$$

Similar calculations give

$$\frac{\partial^2}{\partial x^2} g(x - at) = g''(x - at) \quad \text{and} \quad \frac{\partial^2}{\partial t^2} g(x - at) = a^2 g''(x - at).$$

Therefore

$$u_{tt} - a^2 u_{xx} = a^2 f''(x + at) + a^2 g''(x - at) - a^2 (f''(x + at) + g''(x - at)) = 0.$$