VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 13.1

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Question 1. Determine the domain of the given vector-valued functions.

(a) $\mathbf{r}(t) = \langle \dfrac{1}{t}, \sin t, \sqrt{1-t^2} \rangle$

(b) $\mathbf{r}(t) = \langle t^3, \dfrac{1}{\sin t}, \dfrac{1}{\cos t} \rangle$

Solution 1. Denote $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$.

In (a), we have $\text{Dom}(f) = (-\infty, 0) \cup (0, \infty)$, $\text{Dom}(g) = \mathbb{R}$, $\text{Dom}(h) = [-1, 1]$. Thus, $\text{Dom}(\mathbf{r}) = [-1, 0) \cup (0, 1]$.

In (b), we have $\text{Dom}(f) = \mathbb{R}$, $\text{Dom}(g) = \mathbb{R}$ except for values $t = n\pi$, $n \in \mathbb{Z}$, $\text{Dom}(h) = \mathbb{R}$ except for values $t = \frac{(2n+1)\pi}{2}\pi$, $n \in \mathbb{Z}$. Therefore,

$$\text{Dom}(\mathbf{r}) = \mathbb{R} \backslash \Big(\bigcup_{n=-\infty}^{\infty} \{n\pi\} \cup \bigcup_{n=-\infty}^{\infty} \{\frac{(2n+1)\pi}{2}\} \Big),$$

where $A \backslash B$ means the set A minus the elements in the set B.

Question 2. If $\mathbf{r}$ is a vector-valued function and f a real-valued function, does $f\mathbf{r}$ make sense?

Solution 2. Yes. Write $\mathbf{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$. For any t in the domain of $\mathbf{r}$ and f we have

$$\begin{aligned} f(t)\mathbf{r}(t) &= f(t)\langle r_1(t), r_2(t), r_3(t) \rangle \\ &= \langle f(t)r_1(t), f(t)r_2(t), f(t)r_3(t) \rangle, \end{aligned}$$

where in the second step we used that if c is a scalar then $c\langle x, y, z \rangle = \langle cx, cy, cz \rangle$. Therefore

$$(f\mathbf{r})(t) = f(t)\mathbf{r}(t).$$