VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 12.5

MARCELO M. DISCONZI

Question 1. Does the given line and plan intersect? Where?

$$x = 1 - t,\ y = 1 + 2t,\ z = 3 - t \text{ and } 3x - y + 2z = 5.$$

Solution 1. Plugging $x = 1 - t$, $y = 1 + 2t$, and $z = 3 - t$ into the equation of the plane yields

$$3(1 - t) - (1 + 2t) + 2(3 - t) = 5 \Rightarrow t = -3.$$

Using $t = -3$ into the parametric equations of the line we find $(-4, -5, 6)$.

Question 2. Find the point of intersection of the lines

$$x - 2 - t = 0,\ y - 3 + 2t = 0,\ z - 1 + 3t = 0,$$

$$x - 3 - t = 0,\ y + 4 - 3t = 0,\ z - 2 + 7t = 0.$$

Solution 2. To find the intersection, we set the coordinates of the two lines equal to each other. Denoting by s the parameter on the second line, we find

$$2 + t = 3 + s,\ 3 - 2t = -4 + 3s,\ 1 - 3t = 2 - 7s.$$

All three equations must be satisfied for an intersection to exist. Solving the first two equations gives $t = 2$ and $s = 1$. We verify that these values also satisfy the third equation. Using $t = 2$ on the first line (or $s = 1$ on the second line) produces $(4, -1, -5)$.

Remark 1. In problem 2, a common mistake is not to relabel the parameter in the second line and write

$$2 + t = 3 + t,\ 3 - 2t = -4 + 3t,\ 1 - 3t = 2 - 7t.$$

This is wrong since "t" is a placeholder for different parameters in the two equations.

Remark 2. In problem 2, a common mistake is to forget to verify that the third equation is satisfied. I.e., if we solve the first two equations and find t and s, it is still possible that the values found do not satisfy the third equation, in which case the lines do not intersect.