VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

Examples of section 14.3

Question 1. If f and g are twice differentiable functions of a single variable, show that the function

$$u(x,t) = f(x+at) + g(x-at)$$

is a solution of the wave equation

$$u_{tt} - a^2 u_{xx} = 0,$$

where a is a constant.

Solution 1. Using the chain rule for single variable functions,

$$\frac{\partial}{\partial x} f(x+at) = f'(x+at)\frac{\partial(x+at)}{\partial x} = f'(x+at).$$

Taking another derivative with respect to x,

$$\frac{\partial^2}{\partial x^2} f(x+at) = \frac{\partial}{\partial x} f'(x+at) = f''(x+at)\frac{\partial(x+at)}{\partial x} = f''(x+at).$$

Similarly

$$\frac{\partial}{\partial t} f(x+at) = f'(x+at)\frac{\partial(x+at)}{\partial t} = af'(x+at),$$

and taking another derivative with respect to t,

$$\frac{\partial^2}{\partial t^2} f(x+at) = \frac{\partial}{\partial t}(af'(x+at)) = af''(x+at)\frac{\partial(x+at)}{\partial t} = a^2 f''(x+at).$$

Similar calculations give

$$\frac{\partial^2}{\partial x^2} g(x-at) = g''(x-at) \text{ and } \frac{\partial^2}{\partial t^2} g(x-at) = a^2 g''(x-at).$$

Therefore

$$u_{tt} - a^2 u_{xx} = a^2 f''(x+at) + a^2 g''(x-at) - a^2(f''(x+at) + g''(x-at)) = 0.$$