

VANDERBILT UNIVERSITY

MATH 2300 – MULTIVARIABLE CALCULUS

*Examples of section 12.6*

**Question 1.** Sketch the quadric

$$2x^2 + 4x - y^2 + 6y - z + 4z = 5.$$

**Solution 1.** Completing the square, we see that the equation can be written as

$$z - 2 = 2(x + 1)^2 - (y - 3)^2.$$

Setting  $u = x + 1$ ,  $v = y - 3$ , and  $w = z - 2$ , we have

$$w = 2u^2 - v^2,$$

Which we recognize as a hyperbolic paraboloid in the variables  $(u, v, w)$ . Since  $(u, v, w)$  is obtained from  $(x, y, z)$  by a translation (the origin in the  $(u, v, w)$  system corresponds to the point  $(-1, 3, 2)$  in the original  $(x, y, z)$  variables), the original equation is also a hyperbolic paraboloid.

A few computer generated plots, from different angles, are given below.

