VANDERBILT UNIVERSITY
MATH 198 —METHODS OF ORDINARY DIFFERENTIAL EQUATIONS
PRACTICE MIDTERM.

Question 1. For each equation below, identify the unknown function, classify the equation as linear or non-linear, and state its order.

(a) $y\dfrac{dy}{dx} + \dfrac{y}{x} = 0.$

(b) $x'''' + \cos t\, x' = \sin t.$

(c) $y''' = -\cos y\, y'.$

Question 2. Solve the following initial value problems.

(a) $y' = \dfrac{y-1}{x+3}$, $y(-1) = 0.$

(b) $y' = e^{-x} - 4y$, $y(0) = \dfrac{4}{3}.$

Question 3. Solve the following differential equations.

(a) $y' = \dfrac{2y}{x} - x^2y^2.$

(b) $x' = \dfrac{x^2 + t\sqrt{t^2+x^2}}{tx}.$

(c) $y' = \sin(x-y).$

(d) $y' = \dfrac{\cos y \cos x + 2x}{\sin y \sin x + 2y}.$

Question 4. Find the general solution of the given differential equation.

(a) $y'' + 8y' - 14y = 0.$

(b) $y'' + 8y' - 9y = 0.$

(c) $t^2y'' + 5y = 0$, $t > 0.$

Question 5. Give the form of the particular solution for the given differential equations. You do not have to find the values of the constants of the particular solution.

(a) $y'' + 2y' - 3y = \cos x$.

(b) $y'' + 4y = 8 \sin 2t$.

(c) $y'' - 2y' + y = e^t \cos t$.

(d) $y'' - y' - 12y = 2t^6 e^{-3t}$.

Question 6. Verify that the given functions are two linearly independent solutions of the corresponding homogeneous equation. Then, find a particular solution solving the non-homogeneous problem.

(a) $x^2 y'' - 2y = 3x^2 - 1$, $x > 0$, $y_1 = x^2$, $y_2 = x^{-1}$.

(b) $(1 - x)y'' + xy' - y = \sin x$, $0 < x < 1$, $y_1 = e^x$, $y_2 = x$.

Question 7. Show that the problem

$$3y' - x^2 + xy^3 = 0,\ y(1) = 6,$$

has a unique solution defined in some neighborhood of $x = 1$.

URL: `http://www.disconzi.net/Teaching/MAT198-Spring-14/MAT198-Spring-14.html`