VANDERBILT UNIVERSITY
MATH 196 — DIFFERENTIAL EQUATIONS WITH LINEAR ALGEBRA
EXAMPLES OF SECTION 3.5.

Question 1. Find the inverse of

$$A = \begin{bmatrix} 3 & 5 & 6 \\ 2 & 4 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

SOLUTIONS.

1. Write

$$\begin{bmatrix} 3 & 5 & 6 & \vdots & 1 & 0 & 0 \\ 2 & 4 & 3 & \vdots & 0 & 1 & 0 \\ 2 & 3 & 5 & \vdots & 0 & 0 & 1 \end{bmatrix}$$

and apply Gauss-Jordan elimination.

$$\begin{bmatrix} 3 & 5 & 6 & \vdots & 1 & 0 & 0 \\ 2 & 4 & 3 & \vdots & 0 & 1 & 0 \\ 2 & 3 & 5 & \vdots & 0 & 0 & 1 \end{bmatrix} \underbrace{L_1 \leftarrow}_{\sim} L_1 - L_2 \begin{bmatrix} 1 & 1 & 3 & \vdots & 1 & -1 & 0 \\ 2 & 4 & 3 & \vdots & 0 & 1 & 0 \\ 2 & 3 & 5 & \vdots & 0 & 0 & 1 \end{bmatrix} \underbrace{L_2 \leftarrow}_{\sim} L_2 - L_3$$

$$\begin{bmatrix} 1 & 1 & 3 & \vdots & 1 & -1 & 0 \\ 0 & 1 & -2 & \vdots & 0 & 1 & -1 \\ 2 & 3 & 5 & \vdots & 0 & 0 & 1 \end{bmatrix} \underbrace{L_3 \leftarrow}_{\sim} L_3 - 2L_1 \begin{bmatrix} 1 & 1 & 3 & \vdots & 1 & -1 & 0 \\ 0 & 1 & -2 & \vdots & 0 & 1 & -1 \\ 0 & 1 & -1 & \vdots & -2 & 2 & 1 \end{bmatrix}$$

$$\underbrace{L_3 \leftarrow}_{\sim} L_3 - L_2 \begin{bmatrix} 1 & 1 & 3 & \vdots & 1 & -1 & 0 \\ 0 & 1 & -2 & \vdots & 0 & 1 & -1 \\ 0 & 0 & 1 & \vdots & -2 & 1 & 2 \end{bmatrix} \begin{matrix} L_2 \leftarrow 2L_3 + L_2 \\ L_2 \leftarrow \underbrace{-3L_3}_{\sim} + L_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & \vdots & 7 & -4 & -6 \\ 0 & 1 & 0 & \vdots & -4 & 3 & 3 \\ 0 & 0 & 1 & \vdots & -2 & 1 & 2 \end{bmatrix} \underbrace{L_1 \leftarrow}_{\sim} L_1 - L_2 \begin{bmatrix} 1 & 0 & 0 & \vdots & 11 & -7 & -9 \\ 0 & 1 & 0 & \vdots & -4 & 3 & 3 \\ 0 & 0 & 1 & \vdots & -2 & 1 & 2 \end{bmatrix}$$

So

$$A^{-1} = \begin{bmatrix} 11 & -7 & -9 \\ -4 & 3 & 3 \\ -2 & 1 & 2 \end{bmatrix}.$$