

VANDERBILT UNIVERSITY  
MATH 196 — DIFFERENTIAL EQUATIONS WITH LINEAR ALGEBRA  
EXAMPLES OF SECTIONS 1.5.

**Question 1.** Classify the differential equations below as linear or non-linear and state their order.

- (a)  $y' + \cos(y) = 0$
- (b)  $\frac{d^2x}{dt^2} + \omega^2x = 10 \sin(t)$
- (c)  $yy'' + \frac{1}{x} = y$
- (d)  $e^{\sin x^2} \frac{dy}{dx} + xy = e^{-x}$

**Question 2.** A 100  $\ell$  tank initially contains 10  $kg$  of salt dissolved in 50  $\ell$  of water. Brine containing 1  $kg/\ell$  of salt flows into the tank at the rate 2  $\ell/\text{min}$ , and the well-stirred mixture flows out of the tank at the rate 1  $\ell/\text{min}$ . Write an initial value problem for the amount of salt in the tank.

**SOLUTIONS.**

**Question 1.**

- (a) Non-linear due to  $\cos(y)$ ; first order.
- (b) Linear second order.
- (c) Non-linear due to  $yy''$ ; second order.
- (d) Linear first order.

**Question 2.** Let  $x(t)$  be the amount of salt at time  $t$ , measured in  $kg$ . Then  $\frac{dx}{dt}$  is measured in  $kg/\text{min}$ , and it is given by

$$\frac{dx}{dt} = in - out.$$

We have

$$in = 1 \frac{kg}{\ell} \times 2 \frac{\ell}{min} = 2 \frac{kg}{min},$$

and

$$out = 1 \frac{\ell}{min} \frac{x(t) kg}{V(t) \ell} = \frac{x(t) kg}{V(t) min}.$$

Since  $V(t) = 50 + (2 - 1)t = 50 + t$ , we find

$$\frac{dx}{dt} = 2 - \frac{x}{50 + t}.$$

Since at time zero there were 10  $kg$  of salt, the initial condition is  $x(0) = 10$ . Therefore

$$\begin{cases} \frac{dx}{dt} + \frac{x}{50+t} = 2, \\ x(0) = 10. \end{cases}$$

is the sought initial value problem.