MAT 155B - FALL 12 - EXAMPLES SECTION 8.1

Question 1. Find the length of the given curve:

(a) $x = \frac{y^4}{8} + \frac{1}{4y^2},\ 1 \le y \le 2.$

(b) $y = \sqrt{x - x^2} + \arcsin\sqrt{x}.$

SOLUTIONS.

1a. From $x = \frac{y^4}{8} + \frac{1}{4y^2}$, compute

$$\frac{dx}{dy} = \frac{1}{2}y^3 - \frac{1}{2y^3},$$

so

$$\begin{aligned} 1 + \left(\frac{dx}{dy}\right)^2 &= 1 + \frac{1}{4}y^2 - \frac{1}{2} + \frac{1}{4y^6} \\ &= (\frac{1}{2}y^3 + \frac{1}{2}y^{-3})^2, \end{aligned}$$

and therefore

$$\begin{aligned} L &= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_1^2 \left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right) dy \\ &= \left[\frac{1}{8}y^4 - \frac{1}{4}y^{-2}\right]_1^2 = \frac{33}{16}. \end{aligned}$$

1b. First we need to find the endpoints of the curve. The domain of $\sqrt{x - x^2}$ is given by the values of x such that $x - x^2 \ge 0$, which gives $0 \le x \le 1$. The domain of $\arcsin\sqrt{x}$ is given by the values of x such that $\sqrt{x}$ is in the domain of arcsin, which again gives $0 \le x \le 1$. Therefore the curve starts at $x = 0,\ y = 0$ and ends at $x = 1,\ y = \frac{\pi}{2}$.

Next we compute

$$\begin{aligned} \frac{dy}{dx} &= \frac{1 - 2x}{2\sqrt{x - x^2}} + \frac{1}{2\sqrt{x}\sqrt{1 - x}} \\ &= \frac{2 - 2x}{2\sqrt{x}\sqrt{1 - x}} = \sqrt{\frac{1 - x}{x}}. \end{aligned}$$

Hence,

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1 - x}{x} = \frac{1}{x},$$

and the length of the curve is then given by

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^1 \frac{1}{\sqrt{x}}\, dx = 2\sqrt{x}\Big|_0^1 = 2.$$

URL: `http://www.disconzi.net/Teaching/MAT155B-Fall-12/MAT155B-Fall-12.html`