MAT 155B - FALL 12 - EXAMPLES SECTION 6.6

PROBLEMS.

1. Compute the derivative of $y = \cos^{-1}(\sin^{-1} t)$.
2. Show that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.
3. Show that:

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C.$$

SOLUTIONS.

1. Use the chain rule and the formulas

$$(\cos^{-1} t)' = -\frac{1}{\sqrt{1 - t^2}} \text{ and } (\sin^{-1} t)' = \frac{1}{\sqrt{1 - t^2}},$$

to find

$$(\cos^{-1}(\sin^{-1} t))' = -\frac{(\sin^{-1} t)'}{\sqrt{1 - (\sin^{-1} t)^2}} = -\frac{1}{\sqrt{1 - (\sin^{-1} t)^2}\sqrt{1 - t^2}}.$$

2. Consider a triangle rectangle (i.e., one of its angles is $\frac{\pi}{2}$) with one edge equal to x and hypotenuse equal to 1. Let θ be the angle opposite to x. Then

$$\sin \theta = \frac{x}{1} = x. \tag{1}$$

Since the sum of the angles in a triangle has to be π, the remaining angle is $\frac{\pi}{2} - \theta$, and this angle is adjacent to the side x, hence

$$\cos(\frac{\pi}{2} - \theta) = \frac{x}{1} = x. \tag{2}$$

But (1) gives $\sin^{-1} x = \theta$, whereas (2) gives $\cos^{-1} x = \frac{\pi}{2} - \theta$, so

$$\sin^{-1} x + \cos^{-1} x = \theta + \frac{\pi}{2} - \theta = \frac{\pi}{2},$$

as desired.
3. Write:

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a^2} \int \frac{dx}{1 + \frac{x^2}{a^2}} = \frac{1}{a^2} \int \frac{dx}{1 + \left(\frac{x}{a}\right)^2}.$$

Make the substitution $u = \frac{x}{a}$, so that $dx = a\, du$ and then

$$\frac{1}{a^2} \int \frac{dx}{1 + \left(\frac{x}{a}\right)^2} = \frac{1}{a^2} \int \frac{a\, du}{1 + u^2} = \frac{1}{a} \int \frac{du}{1 + u^2}.$$

Use $\int \frac{du}{1+u^2} = \arctan u + C$ to get

$$\frac{1}{a} \int \frac{du}{1 + u^2} = \frac{1}{a} \arctan u + C = \frac{1}{a} \arctan \frac{x}{a} + C.$$

URL: `http://www.disconzi.net/Teaching/MAT155B-Fall-12/MAT155B-Fall-12.html`