MAT 155B - FALL 12 - EXAMPLES SECTION 6.2*

1. Find x:

(a) $\ln(3x+11) = 7$ (b) $e^{2x} - 5e^x + 6 = 0$

2. Compute the derivative of the given functions:

(a) $y = \ln(\sin x + x)$ (b) $g(z) = \ln\ln\ln(z)$ (c) $f(y) = e^{-\frac{y^2}{\sigma^2}}$ (d) $y = e^{e^x}$

3. Solve the following inequalities for x:

(a) $\ln(x+3) > 2$ (b) $e^{7x-2} < 9$ (c) $\ln(2x+8) - \ln(x+1) > 0$

4. What is the domain of $\ln\frac{2x+8}{x+1}$?

Solutions.

1a. Take ln on both sides and use $e^{\ln x} = x$ to find

$$e^{\ln(3x+11)} = e^7 \Rightarrow 3x + 11 = e^7 \Rightarrow x = \frac{e^7 - 11}{3}.$$

1b. Factor $e^{2x} - 5e^x + 6 = 0$ as $(e^x - 2)(e^x - 3) = 0$. Hence the solution***s*** are

$$e^x - 2 = 0 \Rightarrow x = \ln 2, \text{ and } e^x - 3 = 0 \Rightarrow x = \ln 3.$$

Remark 1. If we had, say, $e^{2x} - e^x - 6 = 0$, factorization gives $(e^x + 2)(e^x - 3) = 0$ and

$$e^x + 2 = 0 \Rightarrow x = \ln(-2) \text{ and } e^x - 3 = 0 \Rightarrow x = \ln 3.$$

But $\ln(-2)$ is undefined, so in this case the only solution would be $x = \ln 3$.

2a. Use the chain rule and remember that $(\ln x)' = \frac{1}{x}$, so

$$y' = \frac{1}{\sin x + x}(\cos x + 1)$$

2b. Apply the chain rule consecutively to find

$$g'(z) = \frac{1}{\ln\ln(z)}(\ln\ln(z))' = \frac{1}{\ln\ln(z)}\frac{1}{\ln(z)}(\ln(z))' = \frac{1}{\ln\ln(z)}\frac{1}{\ln(z)}\frac{1}{z}.$$

2c. Notice that the variable is y, so σ is a constant. Then $f'(y) = -2\frac{y}{\sigma^2}e^{-\frac{y^2}{\sigma^2}}$.

Remark 2. The function $f(x) = e^{-\frac{x^2}{\sigma^2}}$ is known as Gaussian or Normal Distribution, and it is of extreme importance in statistics and applied sciences; in fact, it is used in almost any application which requires data analysis. To know more about it check a book on Statistics or google it.

2d. Use the chain rule:

$$y' = (e^{e^x})' = e^{e^x}(e^x)' = e^{e^x}e^x = e^{e^x+x}.$$

3a. Since exp is a monotone increasing function:

$$\ln(x+3) > 2 \Rightarrow x+3 > e^2 \Rightarrow x > e^2 - 3.$$

3b. Analogously, since ln is monotone increasing,

$$e^{7x-2} < 9 \Rightarrow 7x - 2 < \ln 9 \Rightarrow x < \frac{\ln 9 + 2}{7}.$$

3c. Write $\ln(2x+8) - \ln(x+1) = \ln\frac{2x+8}{x+1}$, thus

$$\ln\frac{2x+8}{x+1} > 0 \Rightarrow \frac{2x+8}{x+1} > e^0 = 1 \Rightarrow 2x+8 > x+1 \Rightarrow x > -7$$

The condition $x > -7$ is not sufficient, since we need to restrict x to values which are in the domain of the function $f(x) = \ln(2x+8) - \ln(x+1)$; for example, $x = -5$ satisfies $x > -7$ but $f(-5)$ is undefined. Hence we need $2x+8 > 0$, giving $x > -4$, and $x+1 > 0$, giving $x > -1$. Combining $x > -7$, $x > -4$ and $x > -1$ we conclude that $\ln(2x+8) - \ln(x+1) > 0$ is satisfied by $x > -1$.

Remark 3. In the above example, we see that, for example, -5 is not in the domain of $f(x) = \ln(2x+8) - \ln(x+1)$, since $f(-5)$ would give $\ln(-2) - \ln(-4)$, and both these terms are undefined. However, if we write

$$\ln(2x+8) - \ln(x+1) = \ln\frac{2x+8}{x+1}$$

and let $g(x) = \ln\frac{2x+8}{x+1}$ then $g(-5) = \ln\left(\frac{-2}{-4}\right) = \ln\frac{1}{2}$ which *is* well defined. What is going on? The point here is that when one writes a formula like

$$\ln X - \ln Y = \ln\frac{X}{Y}$$

it is assumed that both sides of this expression are well defined, i.e., that X, Y, and $\frac{X}{Y}$ are all in the domain of ln. If that is not the case then the formula cannot be applied.

3d. The domain of $\ln\frac{2x+8}{x+1}$ is the set of x-values such that $\frac{2x+8}{x+1} > 0$. A fraction is positive if both numerator and denominator are positive, i.e.,

$$2x+8 > 0 \text{ and } x+1 > 0, \tag{1}$$

or if both numerator and denominator are negative, i.e.

$$2x+8 < 0 \text{ and } x+1 < 0. \tag{2}$$

Condition (1) gives $x > -4$ and $x > -1$, hence $x > -1$; condition (2) gives $x < -4$ and $x < -1$, hence $x < -4$. Therefore the domain of $\ln\frac{2x+8}{x+1}$ is

$$(-\infty, -4) \cup (-1, \infty).$$

URL: `http://www.disconzi.net/Teaching/MAT155B-Fall-12/MAT155B-Fall-12.html`