MAT 155B - FALL 12 - EXAMPLES SECTION 11.2

Question 1. Determine whether the given series converges or diverges.

(a) $\sum_{n=0}^{\infty}(-1)^n$.

(b) $\sum_{n=0}^{\infty}\frac{3^n}{\pi^n}$.

Question 2. Does

$$\sum_{n=0}^{\infty}\frac{1}{n^2+3n+2}$$

converge? If it does, find its sum.

SOLUTIONS.

1a. The sequence of partial sums is

$$\begin{aligned} S_0 &= 1, \\ S_1 &= 1-1=0, \\ S_2 &= 1-1+1=1, \\ S_3 &= 1-1+1-1=0, \end{aligned}$$

We conclude that $S_N = 1$ when N is even, and $S_N = 0$ when N is odd. Therefore the series diverges.

1b. Since

$$0 \leq \frac{3}{\pi} < 1,$$

this is a geometric series with r positive and $r < 1$. Therefore it converges.

2. Let use partial fractions to write

$$\frac{1}{n^2+3n+2} = \frac{1}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2}.$$

Then

$$A(n+2) + B(n+1) = 1.$$

Plugging $n = -1$ we find $A = 1$, and plugging $n = -2$ we find $B = -1$. Therefore

$$\frac{1}{n^2+3n+2} = \frac{1}{n+1} - \frac{1}{n+2}.$$

Hence,

$$\sum_{n=0}^{\infty}\frac{1}{n^2+3n+2} = \sum_{n=0}^{\infty}\Big(\frac{1}{n+1} - \frac{1}{n+2}\Big).$$

The partial sums are

$$S_0 = 1 - \frac{1}{2},$$
$$S_1 = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3},$$
$$S_2 = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4},$$

continuing, we conclude that this is a telescoping series with

$$S_N = 1 - \frac{1}{N+2}.$$

Therefore

$$\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2} = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \left(1 - \frac{1}{N+2}\right) = 1,$$

i.e., the series converges and its sum is 1.

Remark. We cannot break the series as

$$\sum_{n=0}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \sum_{n=0}^{\infty} \frac{1}{n+1} - \sum_{n=0}^{\infty} \frac{1}{n+2},$$

since both

$$\sum_{n=0}^{\infty} \frac{1}{n+1},$$

and

$$\sum_{n=0}^{\infty} \frac{1}{n+2},$$

diverge.

URL: `http://www.disconzi.net/Teaching/MAT155B-Fall-12/MAT155B-Fall-12.html`