MAT 155B - FALL 12 — SECTIONS 04 AND 13
PRACTICE TEST 3

Question 1. Solve the differential equation

$$y' = 2y(y-2).$$

Question 2. What are the constant solutions of the differential equation

$$y' - (y^2 + y^3)\arctan(y+\pi) = 0\,?$$

Question 3. Show that the function $y(x) = xe^{-2x}$ is a solution of the initial value problem

$$\begin{cases} y'' + 4y' + 4y = 0, \\ y(0) = 0,\ y'(0) = 1. \end{cases}$$

Question 4. Determine whether each of the following sequences converges or diverges. You do not have to determine the limit if the sequence converges.

(a) $a_n = \dfrac{(-1)^n n^4}{3n^4+1}$

(b) $a_n = 1 + \dfrac{\sin\frac{n\pi}{2}\ln n}{n}$

(c) $a_1 = 1,\ a_{n+1} = \dfrac{a_n+9}{2}$

(d) $a_n = \cos\dfrac{(2n+1)\pi}{2}$

Question 4. Find the limit of the sequences below.

(a) $a_n = \dfrac{n}{1+\sqrt{4n^2+1}}$

(b) $a_n = \dfrac{5^n}{n!}$

(c) $a_n = \dfrac{\ln(64n^2+1) - \ln(n^2+n)}{4}$

(d) $a_n = n(1 - e^{\frac{1}{n}})$

Question 5. Determine whether each of the following series converges or diverges. You do not have to compute the sum if the series converges.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{3^n-2^n}$

(c) $\displaystyle\sum_{n=1}^{\infty}\tan^2\left(\frac{1}{n}\right)$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n}{\sqrt{4^n+1}}$

(e) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{e^{(-1)^n\sin n}+n^4}$

(f) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\ln(n+4)}$

Question 6. Determine the sum of the following convergent series:

$$\sum_{n=1}^{\infty}\frac{1}{4n^2-9}$$

Question 7. Find all values of p for which the following series converges:

$$\sum_{n=1}^{\infty}n^p\sin^2\left(\frac{1}{n}\right)$$

Question 8. According to the poem by Ogden Nash,

Big fleas have little fleas,
Upon their backs to bite 'em,
And little fleas have lesser fleas,
And so, ad infinitum.

Assume each flea has exactly one flea which bites it. If the largest flea weighs 0.03 grams, and each flea is $\frac{1}{10}$ the weight of the flea it bites, what is the total weight of all the fleas?

URL: http://www.disconzi.net/Teaching/MAT155B-Fall-12/MAT155B-Fall-12.html