

**MAT 155B - FALL 12 — SECTIONS 04 AND 13
PRACTICE TEST 1**

Question 1. Find a formula for the inverse of the given functions:

(a) $h(x) = 17 + \sqrt[5]{\frac{x}{e}}$

(b) $y = \ln \tan \frac{1}{x}$

(c) $y = \frac{e^x}{1+2e^x}$

Question 2. Find the derivative of the given functions:

(a) $y = \frac{1}{\ln \cos e^x}$

(b) $y = \arctan \sqrt{\frac{1-x}{1+x}}$

(c) $f(x) = \arccos \left(\frac{2+3 \cos x}{3+2 \cos x} \right)$

(d) $f(\theta) = \theta \ln(\arctan \theta) - \arcsin(\sqrt{\sin \theta})$

Question 3. Compute the following indefinite integrals:

(a) $\int \frac{7}{11+2x^2} dx$

(b) $\int \tan x dx$

(c) $\int \frac{1}{x\sqrt{1-(\ln x)^2}} dx$

Question 4. Recently physicists at the European Organization for Nuclear Research have discovered a new type of elementary particle, called the Higgs boson. This discovery will help scientists to understand some of the fundamental properties of matter. If the half life of the Higgs boson is about 3 trillions of a second, given a quantity A_0 of such particles find a formula for the amount remaining after t trillions of a second¹.

Question 5. In a certain population model, the rate of change of the population P is proportional to the square of P plus a positive constant B which depends on the details of the environment. If after 3 weeks the population has doubled in size, find a formula for the population after t weeks.

¹Although treatment of half-life of elementary particles is more subtle than what we studied in class for ordinary elements, here you should follow the same reasoning as in the other half-life problems.

Question 6. In a murder investigation, the temperature of the corpse was 32.5°C at 1:30pm and 30.3°C an hour later. Normal body temperature is 37°C and the temperature of the surroundings was 20°C . When did the murder take place?

Question 7. Compute the following limits.

(a) $\lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 5x}$

(b) $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$

(c) $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 - \sin \theta}{\csc \theta}$

(d) $\lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2}$

(e) $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$

(f) $\lim_{x \rightarrow \infty} x \tan \frac{1}{x}$

(g) $\lim_{t \rightarrow \infty} t^{\frac{\ln 2}{1 + \ln t}}$

(h) $\lim_{x \rightarrow 1} (2 - x)^{\tan(\frac{\pi x}{2})}$