MATH 155A FALL 13
PRACTICE MIDTERM 4.

Question 1. Find the derivative of the following functions.

(a) $f(x) = \displaystyle\int_0^x \sqrt{t + \sqrt{t}}\, dt.$

(b) $f(x) = \displaystyle\int_0^{\sqrt{x}} \frac{z^2}{z^4 + 1}\, dz.$

(c) $f(x) = \displaystyle\int_{\tan x}^{x^2} \frac{1}{2 + t^4}\, dt.$

Question 2. Evaluate the following indefinite integrals.

(a) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx.$

(b) $\displaystyle\int \frac{dx}{\cos^2 x\sqrt{1 + \tan x}}.$

(c) $\displaystyle\int x^3\sqrt{x^2 + 1}\, dx.$

(d) $\displaystyle\int \sin x \cos^4 x\, dx.$

(e) $\displaystyle\int \Big(\int_0^{\sin x} t\, dt \Big) \cos x \sin x\, dx.$

Question 3. Evaluate the following definite integrals.

(a) $\displaystyle\int_1^2 \Big(\frac{1}{x^2} - \frac{4}{x^3} \Big)\, dx.$

(b) $\displaystyle\int_1^9 \frac{3x - 2}{\sqrt{x}}\, dx.$

(c) $\displaystyle\int_0^{\frac{3\pi}{2}} |\sin x|\, dx.$

Question 4. Find the area enclosed by the given curves.

(a) $x = y^4$, $y = \sqrt{2 - x}$, $y = 0$.

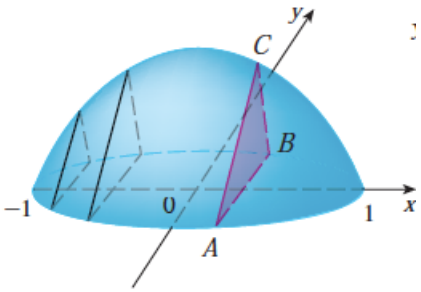
(b) $y = \cos x$, $y = 1 - \cos x$, $0 \leq x \leq \pi$.

(c) $y = \dfrac{1}{4}x^2$, $y = 2x^2$, $x + y = 3$, $x \geq 0$.

Question 5. Using integration, show that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

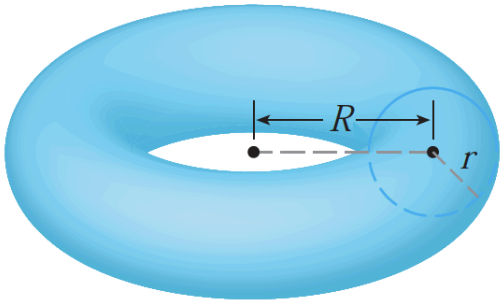
Question 6. Using integration, show that the volume of a cone with a circular base of radius r and height h is $\frac{1}{3}\pi r^2 h$.

Question 7. The picture below shows a solid with a circular base of radius 1. Parallel cross sections perpendicular to the base are equilateral triangles. Find the volume of the solid.



Question 8. Find the volume of the solid S whose base is a circular disk with radius r and parallel cross sections perpendicular to its base are squares.

Question 9. Find the volume of a solid torus, the donut-shaped solid shown in the figure below, of radii r and R.



Question 10. Set up an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

(a) $y = \sqrt{x - 1}$, $y = 0$, $x = 5$ about the $x -$ axis.

(b) $y = x$, $y = 0$, $x = 2$, $x = 4$, about $x = 1$.

(c) $x^2 + 4y^2 = 4$, about $y = 2$.

(d) $x^2 + 4y^2 = 4$, about $x = 2$.

(e) $y^2 - x^2 = 1$, $y = 2$, about the $y -$ axis.

(e) $x = (y - 3)^2$, $x = 4$, about $x = -1$.

Question 11. Prove the Fundamental Theorem of Calculus.

URL: `http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html`