MATH 155A FALL 13
PRACTICE MIDTERM 3.

Question 1. Find all the maxima and minima of the functions on the given intervals.

(a) $f(x) = \cos^2 x - 2\sin x,\ \text{ on } [0, 2\pi]$.

(b) $f(x) = x\sqrt{6-x},\ \text{ on } [-10, 6]$.

(c) $f(x) = 5x^{\frac{2}{3}} - 2x^{\frac{5}{3}},\ \text{ on } (-\infty, \infty)$.

Question 2. Show that the equation $2x - 1 - \sin x = 0$ has exactly one real root.

Question 3. Find the limit or show that it does not exist.

(a) $\displaystyle\lim_{x\to\infty} \frac{x \sin x}{x^2+1}$.

(b) $\displaystyle\lim_{x\to-\infty} \left(x + \sqrt{x^2+2x}\right)$.

(c) $\displaystyle\lim_{x\to\infty} \frac{x^2 - x^4}{3x}$.

(d) $\displaystyle\lim_{x\to-\infty} \frac{2x^5 + x^4 - 3x^2 + 7}{x(2x - 4x^3 - 5x^4)}$.

(e) $\displaystyle\lim_{x\to\infty} \sqrt{x}\sin\frac{1}{x}$.

Question 4. Sketch the graph of the given functions.

(a) $y = \dfrac{1-x}{1+x}$.

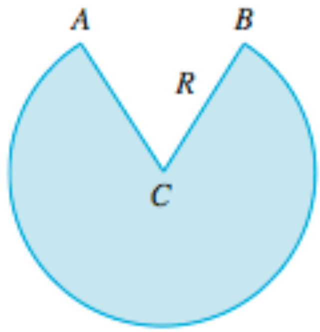
(b) $y = \sqrt{x^2+x} - x$.

(c) $y = \dfrac{\sin x}{2 + \cos x}$.

Question 5. A right circular cylinder is inscribed in a sphere of radius R. Find the largest possible surface area of such cylinder.

Question 6. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$ 10 per square meter. Material for the sides costs \$ 6 per squre meter. Find the cost of materials for the cheapest container.

Question 7. A cone-shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining the edges CA and CB. Find the maximum capacity of such a cup.



Question 8. Find f if

(a) $f''(x) = 6x + \sin x$.

(b) $f'(x) = x^{-\frac{1}{3}}$, $f(1) = 1$, $f(-1) = -1$.

(c) $f'''(x) = \cos x$, $f(0) = 1$, $f'(0) = 2$, $f''(0) = 3$.

Question 9. Use the form of the definition of the integral as a limit of sums to evaluate the integral.

(a) $\displaystyle\int_1^4 (x^2 - 4x + 2)dx$.

(b) $\displaystyle\int_2^4 (x^3 - 1)dx$.

Question 10. Express the integral as a limit of Riemann sums. Do not evaluate the limit.

$$\int_0^{2\pi} x^2 \sin x \, dx.$$

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