

**MATH 155A FALL 13  
PRACTICE MIDTERM 2.**

Question 1. Find  $y'$ .

(a)  $y = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt[5]{x^3}}$ .

(b)  $y = \frac{\tan x}{1 + \cos x}$ .

(c)  $y = x^{\sqrt{21+\pi}} \cos x$ .

(d)  $y = \frac{1}{\sin(x - \sin x)}$ .

(e)  $y = \sin^2(\cos(\sqrt{\sin(\pi x)}))$ .

Question 2. Find an equation for the tangent line and normal line to the curve at the given point.

(a)  $y = (1 + 2x)^2$ ,  $(1, 9)$ .

(b)  $y = \frac{\sqrt{x}}{x + 1}$ ,  $(4, 0.4)$ .

(c)  $x^2 + 2xy - y^2 + x = 2$ ,  $(1, 2)$ .

Question 3. An object of mass  $m$  is shot straight upward from the ground with initial velocity  $v_0$ . Assuming that there is no air resistance, the height  $h$  of the object, measured with respect to the ground level, is given as a function of time, by

$$h(t) = v_0 t - \frac{1}{2} g t^2,$$

where  $t$  is the time, and  $g$  is the value of the gravitational acceleration. Show that the maximum height of the object is

$$h_{max} = \frac{1}{2} \frac{v_0^2}{g}.$$

Suppose that gravity in the planet Krypton is such that its gravitational acceleration is twice that of Earth's. How much faster would the object have to be shot in order to reach the same height?

Question 4. Find  $y''$  by implicit differentiation.

(a)  $x^3 + y^3 = 1$ .

(b)  $\sqrt{x} + \sqrt{y} = 1$ .

Question 5. Two sides of a triangle have lengths 12m and 15m. The angle between them is increasing at a rate of  $2^\circ/\text{min}$ . How fast is the length of the third side increasing when the angle between the sides of fixed length is  $60^\circ$ ?

Question 6. A balloon is raising at a constant speed of 4 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 seconds later?

Question 7. Use linear approximation to estimate  $\frac{1}{4.002}$ .

Question 8. Find the absolute maximum and absolute minimum values of  $f(x) = \frac{x}{x^2-x+1}$  on  $[0, 3]$ .

Question 9. Find the local and absolute maxima and minima of  $x^3 + x^2 - 4x - 4$  on the interval  $[-3, 10]$ .

Question 10. Prove the chain rule.

*URL:* <http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html>