MATH 155A FALL 13
EXAMPLES SECTION 4.3.

Question. Find g' if

$$g(x) = \int_{x^3}^{\cos(\pi x)} t^4 \, dt.$$

Solution. In order to apply the FTC, we need a function that is written in the form

$$\int_a^x f(t) \, dt.$$

Write

$$\begin{aligned} \int_{x^3}^{\cos(\pi x)} t^4 \, dt &= \int_{x^3}^{a} t^4 \, dt + \int_a^{\cos(\pi x)} t^4 \, dt \\ &= -\int_a^{x^3} t^4 \, dt + \int_a^{\cos(\pi x)} t^4 \, dt, \end{aligned}$$

for some fixed number a. In order to compute the derivative of $\int_a^{x^3} t^4 \, dt$, we let $u = x^3$ and use the chain rule:

$$\begin{aligned} \frac{d}{dx} \int_a^{x^3} t^4 \, dt &= \frac{d}{dx} \int_a^{u} t^4 \, dt \\ &= \Big(\frac{d}{du} \int_a^u t^4 \, dt \Big) \frac{du}{dx} \\ &= u^4 \frac{du}{dx} = (x^3)^4 \frac{dx^3}{dx} \\ &= 3x^{14}. \end{aligned}$$

Analogously, letting $v = \cos(\pi x)$,

$$\begin{aligned} \frac{d}{dx} \int_a^{\cos(\pi x)} t^4 \, dt &= \int_a^{v} t^4 \, dt \\ &= \Big(\frac{d}{dv} \int_a^v t^4 \, dt \Big) \frac{dv}{dx} \\ &= v^4 \frac{dv}{dx} = (\cos(\pi x))^4 (-\pi \sin x) \\ &= -\pi \sin x \cos^4(\pi x). \end{aligned}$$

Thus,

$$g'(x) = -3x^{14} - \pi \sin x \cos^4(\pi x).$$

URL: `http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html`