MATH 155A FALL 13
EXAMPLES SECTIONS 2.7 AND 2.8.

Question 1. A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

Question 2. When air expands adiabatically (i.e., without gaining or losing heat), its pressure P and volume V are related by the equation $PV^{1.4} = C$, where C is a constant. Suppose that at a certain instant the volume is 400 cm^3 and the pressure is 80 kPa and is decreasing at a rate of 10 kPa/min. At what rate is the volume increasing at this instant?

Solutions.

1. Consider a triangle with one side A equal to 3 km which represents the distance from the lighthouse to the point P on the shore. Let x be the distance from P to a point Q; A and the line from P to Q meet orthogonally. Let θ be the angle between A and the line B from the lighthouse to Q.

We are given $\frac{d\theta}{dt} = 4 \times 2\pi = 8\pi$ rad/min. We have $\tan\theta = \frac{x}{3}$, or $x = 3\tan\theta$. Hence

$$\frac{dx}{dt} = 3\sec^2\theta \frac{d\theta}{dt} = 24\pi \sec^2\theta.$$

When $x = 1$, the distance from the lighthouse to Q is $\sqrt{3^2 + 1^2} = \sqrt{10}$, so that $\cos\theta = \frac{3}{\sqrt{10}}$. Hence $\sec^2\theta = \frac{10}{9}$, and we obtain

$$\frac{dx}{dt} = \frac{240\pi}{9} = \frac{80\pi}{3} \text{ km/min}.$$

2. Differentiate $PV^{1.4} = C$ to get

$$1.4PV^{0.4}\frac{dV}{dt} + V^{1.4}\frac{dP}{dt} = 0 \Rightarrow \frac{dV}{dt} = -\frac{V}{1.4P}\frac{dP}{dt}.$$

When $V = 400$, $P = 80$ and $\frac{dP}{dt} = -10$. Hence

$$\frac{dV}{dt} = -\frac{400}{1.4 \times 80} \times (-10) = \frac{250}{7} \text{ cm}^3\text{/min}.$$

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