MATH 155A FALL 13
EXAMPLES SECTION 2.3.

1. Compute $f'(x)$ if
(a) $f(x) = \frac{x^2-4}{x^2+4}$.
(b) $f(x) = 3x^5 - 2x^2 + 1$.

2. Find the points on the graph of $\frac{x^2-4}{x^2+4}$ where the tangent line is horizontal.

Solutions.
(1a) Compute

$$\left(\frac{x^2-4}{x^2+4}\right)' = \frac{(x^2+4)(x^2-4)' - (x^2-4)(x^2+4)'}{(x^2+4)^2}$$
$$= \frac{(x^2+4)(2x-0) - (x^2-4)(2x+0)}{(x^2+4)^2}$$
$$= \frac{2x^3+8x-2x^3+8x}{(x^2+4)^2}$$
$$= \frac{16x}{(x^2+4)^2}.$$

(1b) Compute

$$(3x^5 - 2x^2 + 1)' = (3x^5)' - (2x^2)' + 1'$$
$$= 3(x^5)' - 2(x^2)' + 0$$
$$= 3 \cdot 5x^4 - 2 \cdot 2x$$
$$= 15x^4 - 4x.$$

(2) The tangent line is horizontal when $f'(x) = 0$. We computed the derivative of $f(x) = \frac{x^2-4}{x^2+4}$ above, thus

$$\left(\frac{x^2-4}{x^2+4}\right)' = \frac{16x}{(x^2+4)^2} = 0$$
$$\Rightarrow 16x = 0 \Rightarrow x = 0.$$

Since $f(0) = \frac{0^2-4}{0^2+4} = -1$, the point is $(0, -1)$.

URL: `http://www.disconzi.net/Teaching/MAT155A-Fall-13/MAT155A-Fall-13.html`