Stony Brook University.

MAT 123 — Precalculus, Summer 2011.

Practice Final.

NAME: ID:

Question 1. When is a function one-to-one? What is the inverse of a function? What is the horizontal line test? What is it used for and how? Give examples.

Question 2. Given the equations below identify if they define y as a function of x. For those which are functions, say whether their inverse exists; find their inverse when possible.

(a) $y = \frac{1}{x^2-3}$ (b) $y^{\frac{1}{2}}+\frac{1}{x} = 1$ (c) $\log_3 y - \frac{1}{3x+\sqrt{x}} = 0.$ (d) $\cos(y) - x = 0$ (e) $y - x^2 = 0,\ x \geq 0.$

Question 3. A 400-room hotel can rent every one of its rooms at \$120 per room. For each \$1 increase in rent, two fewer rooms are rented.

(a) Express the number of rooms rented as a function of the rent.

(b) Express the hotel's revenue as a function of the rent.

Question 4. Given a cylinder with circular base and fixed volume, express its surface area as a function of its height.

Question 5. Graph:

(a) $y = \frac{1}{(x+2)^2} - 1$ (b) $y = \frac{-2x^3}{x^2+1}$ (c) $y = \frac{4x^2-16x+14}{2x-3}$ (d) $y = \frac{x^3-4x^2-12x}{x^2+4x+3}$ (e) $y = \frac{2x}{x^4-13x^2+36}$

Question 6. Solve for x:

(a) $e^{12-5x} - 7 = 123$ (b) $3+4\ln(2x) = 15$ (c) $\log_3(x-1) - \log_3(x+2) = 2$ (d) $e^{2x} - e^x - 6 = 0$

(e) $2\log_4(2x+1) = \log_4(x-3) + \log_4(x+5)$ (f) $3^{x+4} = 7^{2x-1}$

Question 7. A radioactive substance decays from 100 grams to 82 grams in 1,000 years. Assuming continuous-exponential decay, determine its half-life. How many years will it take to decay to 10 grams?

Question 8. Compute:

(a) $\sin\frac{\pi}{6} + \tan^2\frac{\pi}{3}$ (b) $\sec^2\frac{\pi}{5} - \tan^2\frac{\pi}{5}$ (c) $\sin 240°$ (d) $\tan 120°$ (e) $\sec\frac{7\pi}{4}$ (f) $\sin\frac{22\pi}{3}$

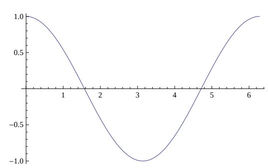
(g) $\sin 495°$ (h) $\cos(-\frac{35\pi}{6})$ (i) $\tan\frac{13\pi}{4}$

Question 9. Graph:

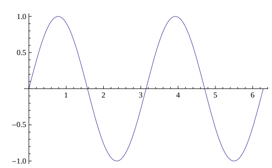
(a) $y = 3\sin(4x)$ (b) $y = \frac{1}{2}\sin\frac{\pi}{3}x$ (c) $y = 2\sin(x-\pi)$ (d) $y = -3\cos(x+\pi)$ (e) $y = \frac{3}{2}\cos(\frac{\pi}{4}-2x)$

(f) $y = -3\sin(\frac{\pi}{3}x - 3\pi)$ (g) $y = -\tan(x - \frac{\pi}{4})$ (h) $y = 2\tan(-x - \frac{\pi}{2})$

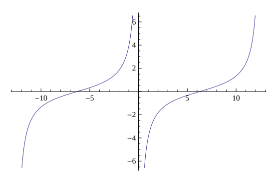
Question 10. Write a function that best describes each graph below.



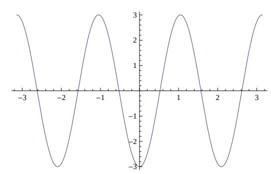
(a)



(b)



(c)



(d)

Question 11. Compute:

(a) $\sin^{-1} 1$ (b) $\cos^{-1} 1$ (c) $\cos^{-1}(-\frac{1}{2})$ (d) $\tan^{-1}(-\frac{\sqrt{3}}{3})$ (e) $\cos(\sin^{-1}\frac{\sqrt{2}}{2})$

(f) $\sin^{-1}(\sin\frac{2\pi}{3})$ (g) $\cos(\tan^{-1}\frac{5}{2})$ (h) $\sec(\sin^{-1}\frac{1}{x})$

Question 12. Find all solutions of the equations below:

(a) $\cos x = \frac{1}{2}$ (b) $\sqrt{2}\sin 4x = 1$ (c) $\sqrt{3}\tan x - 1 = 0$

Question 13. Solve each equation on the interval $[0, 2\pi)$:

(a) $\sin 3x = 1$ (b) $\cos^2(2x) - 2\cos(2x) = 3$ (c) $2\cos^2 x - \sin x = 1$