VANDERBILT UNIVERISTY
MATH 8110 — THEORY OF PARTIAL DIFFERENTIAL EQUATIONS
HW 6

Unless stated otherwise, the notation below is as in class.

1. Problems

Problem 1. Show that it is not true that if $k+\gamma < m+\delta$, then $C^{m+\delta}(\bar{\Omega}) \subset C^{k+\gamma}(\bar{\Omega})$.

Problem 2. Show that $C^{k,\alpha}(\bar{\Omega}) \subset C^{k,\beta}(\bar{\Omega}), \beta < \alpha$.

2. Solutions

Solution 1. This is done in Section 9.5 of the class notes.

Solution 2. We have for $|\gamma| \leq k$,

$$\sup_{\substack{0<|x-y|<1 \\ x,y\in\Omega}} \frac{|D^\gamma u(x) - D^\gamma u(y)|}{|x-y|^\beta} \leq \sup_{\substack{x\neq y \\ x,y\in\Omega}} \frac{|D^\gamma u(x) - D^\gamma u(y)|}{|x-y|^\alpha}.$$

We also have

$$\sup_{\substack{|x-y|\geq 1 \\ x,y\in\Omega}} \frac{|D^\gamma u(x) - D^\gamma u(y)|}{|x-y|^\beta} \leq 2 \sup_{x\in\Omega} |D^\gamma u(x)|,$$

which implies the result.