MATH 2610, EXAMPLES OF SECTION 4.4

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Question 1. Write the form of the particular solution for the equations below (you do not have to find the values of the constants).
(a) $y'' + 9y = 2\cos(3x) + 3\sin(3x)$.
(b) $y'' + 9y = 2x^2 e^{3x} + 5$.

SOLUTIONS.
1a. The homogeneous equation is

$$y'' + 9y = 0,$$

with characteristic equation

$$\lambda^2 + 9 = 0,$$

whose roots are $\pm 3i$. Hence

$$y_1 = \cos(3x),\ \ y_2 = \sin(3x),$$

are solutions of the homogeneous equation. Given the form of $f(x)$, we look for

$$y_p = x^s\big(A\cos(3x) + B\sin(3x)\big).$$

Since $\cos(3x)$ and $\sin(3x)$ are solutions of the homogeneous equation, we need $s = 1$, so

$$y_p = x\big(A\cos(3x) + B\sin(3x)\big).$$

1b. The homogeneous equation is

$$y'' + 9y = 0,$$

with characteristic equation

$$\lambda^2 + 9 = 0,$$

whose roots are $\pm 3i$. Hence

$$y_1 = \cos(3x),\ \ y_2 = \sin(3x),$$

are solutions of the homogeneous equation. Given the form of $f(x)$, we look for

$$y_p = x^s A + x^r\big(Bx^2 + Cx + D\big)e^{3x}.$$

Since there is no repetition with the solutions of the homogeneous equation, $r = s = 0$ and

$$y_p = A + \big(Bx^2 + Cx + D\big)e^{3x}.$$