MATH 2610, EXAMPLES OF SECTIONS 1.1 AND 1.2

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Question 1. Classify the differential equations below as linear or non-linear and state their order.
(a) $y' + \cos(y) = 0$
(b) $\frac{d^2x}{dt^2} + \omega^2 x = 10\sin(t)$
(c) $yy'' + \frac{1}{x} = y$
(d) $e^{\sin x^2}\frac{dy}{dx} + xy = e^{-x}$

Question 2. A $100\,\ell$ tank initially contains $10\,kg$ of salt dissolved in $50\,\ell$ of water. Brine containing $1\,kg/\ell$ of salt flows into the tank at the rate $2\,\ell/\min$, and the well-stirred mixture flows out of the tank at the rate $1\,\ell/\min$. Write an initial value problem for the amount of salt in the tank.

SOLUTIONS.

Question 1.

(a) Non-linear due to $\cos(y)$; first order. (b) Linear second order. (c) Non-linear due to yy''; second order. (d) Linear first order.

Question 2. Let $x(t)$ be the amount of salt at time t, measured in kg. Then $\frac{dx}{dt}$ is measured in kg/min, and it is given by

$$\frac{dx}{dt} = in - out.$$

We have

$$in = 1\frac{kg}{\ell} \times 2\frac{\ell}{min} = 2\frac{kg}{min},$$

and

$$out = 1\frac{\ell}{min}\frac{x(t)\,kg}{V(t)\,\ell} = \frac{x(t)}{V(t)}\frac{kg}{\min}.$$

Since $V(t) = 50 + (2-1)t = 50 + t$, we find

$$\frac{dx}{dt} = 2 - \frac{x}{50+t}.$$

Since at time zero there were $10\,kg$ of salt, the initial condition is $x(0) = 10$. Therefore

$$\begin{cases} \frac{dx}{dt} + \frac{x}{50+t} = 2, \\ x(0) = 10. \end{cases}$$

is the sought initial value problem.