Correlation functions

0. Notation and pre-requisits

Our convention for the metric is $+---$
Recall that the four-momentum is $p^\mu = \left(\frac{E}{c}, \vec{p}\right)$
where $E(\vec{p}) = \sqrt{c^2\vec{p}^{\,2} + M^2c^4}$ where M is the mass of the particle (Ry, 27). Introducing $\vec{k} = \frac{\vec{p}}{\hbar}$ the energy can be rewritten as $E(\vec{k}) = c\hbar\sqrt{\vec{k}^2 + \frac{M^2c^2}{\hbar^2}}$
$= c\hbar\sqrt{\vec{k}^2 + m^2}$ where $m = \frac{Mc}{\hbar}$; or yet
$E(\vec{k}) = c\hbar\,\omega(\vec{k})$ with $\omega(\vec{k}) = \sqrt{\vec{k}^2 + m^2}$. (St, 8).
We are going to work in natural units: $\hbar = c = 1$, so
so the four momentum can be identified with the four-vector $k^\mu = (\omega, \vec{k})$. In particular notice the Lorentz invariant quantity $k^2 = k^\mu k_\mu = \omega^2 - \vec{k}^2 = m^2$

When dealing with the operator approach, most of the time we will be working ~~in~~ in the Heisenberg picture. Recall that in the ~~[illegible]~~ Heisenberg picture states are time independent and operators are time dependent:

$$-i\hbar \frac{dA(t)}{dt} = [H, A(t)] \quad , \quad \frac{d}{dt}|\psi(t)\rangle = 0$$

$$A(t) = e^{\frac{itH}{\hbar}} A(0)\, e^{-\frac{itH}{\hbar}} \quad , \quad |\psi(t)\rangle = |\psi(0)\rangle$$

while in the Schrödinger picture states are time dependent and operators are time-independent:

$$\frac{dA(t)}{dt} = 0 \, , \quad i\hbar \frac{d|\psi(t)\rangle}{dt} = H|\psi(t)\rangle$$

$$A(t) = A(0) \quad , \quad |\psi(t)\rangle = e^{-\frac{itH}{\hbar}} |\psi(0)\rangle$$

and in both cases H does not depend explicitly on time. (St, 502)

Notice that since in the Heisenberg picture $|\psi(t)\rangle_H = |\psi(0)\rangle_H$ (where H stands for Heisenberg) and $|\psi(0)\rangle_S = |\psi(0)\rangle_H$ (where S stands for Schrödinger) the last of the equations for the Schrödinger picture gives

$$|\psi(t)\rangle_S = e^{-\frac{itH}{\hbar}} \, |\psi(t)\rangle_H$$

In particular, when dealing with eigenstates of the position operator ($|q(t)\rangle_S$ in the Schrödinger picture), we see expressions such as $|q,t\rangle_H$
But in the Heisenberg picture states are time-independent, so how do we interpret this?
t here is to be interpreted as a <u>label</u>: it tells us that $|q\,t\rangle$ is an eigenstate of the operator $Q(t)$ (or $Q_H(t)$ if we want to stress that this is the position operator in the Heisenberg picture at time t) but not of $Q(t')$ if $t \neq t'$. Indeed (using $|\ \rangle_S = e^{(\ )} |\ \rangle_H$)

$$Q(t)\,|q,t'\rangle_H = Q(t)\, e^{\frac{it'H}{\hbar}} \, |q(t')\rangle_S$$

$$\shortparallel$$

$$e^{\frac{itH}{\hbar}} Q(0)\, e^{-\frac{itH}{\hbar}}$$

So

$$Q(t)\,|q,t'\rangle_H = e^{\frac{itH}{\hbar}}\, Q(0)\, e^{\frac{i(t'-t)H}{\hbar}}\, |q(t')\rangle_S$$

If $t = t'$ then we have:

$$Q(t)\,|q,t'\rangle_H = e^{\frac{itH}{\hbar}}\, Q(0)\, |q(t')\rangle_S = e^{\frac{itH}{\hbar}}\, Q(0)\, |q(t)\rangle_S$$

Now $Q(0) = Q_S$ is the position operator in Schrödinger picture, so $Q_S\,|q(t)\rangle_S = q(t)\,|q(t)\rangle_S$ ($q(t)$ = eigeinvalue of $|q(t)\rangle_S$; it is not an operator), so

$$Q(t)\,|q,t\rangle_H = q(t)\,\underbrace{e^{\frac{itH}{\hbar}}\,|q(t)\rangle_S}_{=\,|q,t\rangle_H} = q(t)\,|q,t\rangle_H$$

as claimed; and the above computations also show that we don't get an eigenvector for $t \neq t'$.