CORRELATION FUNCTIONS IN QUANTUM FIELD THEORY

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Abstract. The basic ideas and concepts of quantum field theory are discussed with the intent of making physics books and papers on the subject more accessible to a mathematical audience. The focus is on correlation functions for the scalar field: what they are, how to compute them, their Feynman diagrams and renormalization properties.

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2. About these notes

These notes grew out of a series of informal meetings involving graduate students and professors of the Mathematics Department of Stony Brook University. Our goal was to understand the basic concepts and tools of quantum field theory (QFT).

I initially volunteered to conduct the discussions because of my familiarity with the topic. Over time more people joined us, including students and faculty from the Stony Brook Physics Department, and parts of this material had been presented at the RTG Seminar in Geometry and Physics at Stony Brook University. These notes therefore contain inputs and suggestions from several people, and in some sense they should be considered as the product of a collective effort toward a better understanding of the subject.

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There is a great deal of interest in QFT among mathematicians, probably due to deep connections between different mathematical structures encoded in QFT. This has given rise to a substantial literature covering its rigorous formulation, such as the famous AMS two volume "QFT and Strings: A Course for Mathematics", edited by Deligne *et al.*, which added to more traditional and classical rigorous treatments of the subject, such as Glimm and Jaffe's "Quantum Physics — A Functional Integral Point of View".

However, the mathematical treatment of QFT requires a wide spectrum of high mathematics which makes such rigorous presentations almost detached from a typical physicist's approach. As a result, even after going over such sophisticated formulations of QFT, mathematicians are still unable to read the physics literature. In fact, members of our group have found it difficult at best to understand the QFT materials authored by physicists.

My goal here is to explain the main concepts of QFT to a mathematical audience, without giving a *mathematical treatment* of the subject. In other words, my plan is to present QFT in the spirit of physics books, pausing and explaining when necessary the possible mathematical interpretation of the objects and procedures involved.[1] For example, when introducing the canonical quantization of fields, it will be done by a map $\varphi : \mathbb{R}^4 \to \{\text{Operators in some Hilbert space}\}$, as it is usually done by physicists. Then I will point out the mathematical problems with this formulation, such as the fact that such functions actually *do not exist*, being rather (operator valued) distributions — what prevents any meaningful interpretation of pointwise expressions such as $\varphi(x)$. However, such pointwise expressions appear all the time in the physics literature, and so a discussion about how to meaningfully interpret them is carried out. With that said, the subject is not pursued further, i.e., we will not go about proving the existence of such distributions or so on, rather referring to the relevant literature.

In order to make the material more friendly to mathematicians, ideally one would try to use a mathematical style with "definitions" and "theorems", even though most of the time the objects involved are ill-defined from a mathematical perspective. The only reason why such approach has not been taken is that I was planning on typesetting the material, on which occasion such adjustments would be introduced. Although I still intend to do so *eventually*, there is no prospect that this may happen any time soon. But as several people who have had access to these notes told me that they benefited from them, I found useful to make the notes available, even if in the present raw and somewhat unpleasant format.

Despite of the targeted audience, I'll also take the opportunity to motivate certain topics from a physical perspective — in any case, I believe that mathematicians who really want to learn some QFT do have some appreciation for its physical aspects, and therefore would not shy away from such discussions. I'll do so even when a certain topic can be justified purely on mathematical grounds. For example, physicists seem to be concerned most of the time with correlation functions. In the mathematical literature we learn that these correlation functions characterize the theory, in the sense that it can be "reconstructed" from them. But, of course, this is not the primary reason why physicists are interested in such functions. They want to know the correlation functions because they are closely related to quantities which can be measured directly in a laboratory.

Despite being an amalgamation of several sources, I have tried my best to present the subject in a coherent and logical fashion — what didn't prevent me from making incessant references to those sources along the text. Needless to say, there is no attempt to originality, and some parts are copied

[1]In this sense, the intent of this presentation resembles a lot to Folland's "Quantum Field Theory, A Tourist Guide for Mathematicians". In fact, the only reason his nice book is not mentioned even once in these notes is because it was published *after* I had written the material.

directly from one or more references. I have also made a significant effort to maintain the text as self contained as possible (hence the inclusion of several appendices), although knowledge of Quantum Mechanics is certainly assumed, and some parts might in fact be hard to grasp at first for those who have never had any previous contact, even if superficial, with QFT. Finally, I was careful enough to keep track of all factors of $\frac{1}{2}$, negative signs and etc, *specially* when it comes to renormalization, since then such numerical terms are crucial for the correct cancellations.