REAL ANALYSIS, HW 1

VANDERBILT UNIVERSITY

This homework is a warm-up for the course and is optional.

The notation used below follows the one used in class and should be self-explanatory.

Question 1. Prove the properties:

(a) $(X \cup Y) \times Z = (X \times Z) \cup (Y \times Z)$

(b) $(X \cap Y) \times Z = (X \times Z) \cap (Y \times Z)$

(c) $(Y \cup Z)^c = Y^c \cap Z^c$

(d) $(Y \cap Z)^c = Y^c \cup Z^c$

(e) $f^{-1}(X \cup Z) = f^{-1}(X) \cup f^{-1}(Z)$

(f) $f^{-1}(X \cap Z) = f^{-1}(X) \cap f^{-1}(Z)$

Question 2. Prove the statements:

(a) Let X be an infinite subset of $\mathbb{Z}_+$. Then X is denumerable, and in fact there exists a unique enumeration of X, namely, $\{k_1, k_2, \dots\}$, such that

$$k_1 < k_2 < \cdots < k_n < k_{n+1} < \dots$$

(b) Every infinite set contains a denumerable subset.

(c) Let X be a denumerable set, and $f : X \to Y$ a surjective map. Then Y is denumerable.

(d) Let $\{D_1, D_2, \dots\}$ be a sequence of denumerable sets. Then

$$\bigcup_{i=1}^{\infty} D_i$$

is denumerable.

Question 3. Give an example of a partially ordered set without a maximal element.

Question 4. Show that a maximal element need not to be unique. A greatest element, however, if it exists, is unique.

Question 5. Prove Zorn's lemma (of course, we could take it as an axiom, but you know what we mean, right?).