VANDERBILT UNIVERSITY

MATH 3120 – INTRO DO PDES

Study guide for the second test

Below are topics and ideas that should be emphasized in your studies for the second test.

- Understand what a Fourier series is, including the difference between a Fourier series for a function and the function itself.
- Know how to derive the formulas for the coefficients of the Fourier series both in the cases $[-L, L]$ and $[0, L]$. You will not be asked to compute the formulas involving integrals of $\sin \frac{n\pi x}{L}$ and $\cos \frac{m\pi x}{L}$, but you should memorize and know how to use them.
- Be ready to state and apply the convergence theorems for Fourier series discussed in class.
- Understand the general philosophy of the separation of variables method.
- Know the difference between formal solutions and actual solutions.
- Be ready to derive formal solutions to the wave and heat equations in one spatial dimension under the different types of boundary conditions studied in class and in the homework.
- Understand how to use D'Alembert's formula to prove that a given formal solution to the wave equation on $[0, L]$ is an actual solution. (This was done in class, see the class notes.)
- Understand how to prove that formal solutions to the heat equation give rise to actual solutions. (Understand the argument in the solutions to homework 4.)

The solution to the Schrödinger equation is too long to be asked in the test. However, you may be asked to reproduce some of the arguments/calculation carried out in the solution derived in class, such as:

- Understand how the constants that appear in the separation of variables for the Schrödinger equation are determined. In particular, understand how some of those constants are determined upon some convergence requirement of series solutions.
- Understand the change of variables carried out in the solution to the Schrödinger equation.
- Understand why the constant E has to be real and negative.
- Understand how the conditions $\int_{\mathbb{R}^3} |\Psi(x,t)|^2 \, dx < \infty$ and $\int_{\mathbb{R}^3} |\Psi(x,t)|^2 \, dx = 1$ are related, and their role in the arguments related to convergence of series solutions in the study of the Schrödinger equation.

Finally, make sure to review the class material available, in particular:

- Review the posted homework solutions.
- Review the posted class notes.
- Review the notes on the Schrödinger equation.