VANDERBILT UNIVERSITY

MATH 3120 – INTRO DO PDES

Study guide for the final exam

General information about the final exam:

- The final exam is cumulative. However, there will be an emphasis on topics covered after the second test, including: the fundamental solution to the Laplacian, solutions to Poisson's equation in $\mathbb{R}^n$, Green's function, the wave equation in $\mathbb{R}^n$.

The following topics will not be covered in the final exam:

- The Schrödinger equation.
- Separation of variables.
- Laplace transform.

Below are topics and ideas that should be emphasized in your studies for the final exam.

- Understand the statement of the theorem about existence and uniqueness of solutions to first-order PDEs via the method of characteristics. Be ready to state the theorem, know how to determine if a given initial data is characteristic, and how to apply the theorem.
- Know how to find explicit solutions to PDEs using the method of characteristics.
- Know the derivation of D'Alembert's formula.
- Be ready to state the convergence theorems about Fourier series discussed in class. Know how to use these theorems.
- Know how to use the Fourier transform to solve PDEs.
- Understand what the fundamental solution to the Laplacian is. Know how to state a formula for a solution to Poisson's equation in $\mathbb{R}^n$ in terms of the fundamental solution and the given data.
- Understand what the Green function is. Know how to derive a formula for a solution of a boundary-value problem for Poisson's equations in terms of the Green's function and the given data.
- Know how to prove conservation of energy for the wave equation.
- Know how to prove the finite speed of propagation property for the wave equation.
- Know how to prove uniqueness of solutions to the initial-value problem for the wave equation.
- Know Duhamel's principle.

The proof of existence of solutions for Poisson's equation in $\mathbb{R}^n$ presented in class is too long to be asked in the final exam. However, you may be asked to reproduce parts of the argument. In particular, understand:

- The computations involving derivatives of the fundamental solution.
- The arguments where integrals have to be split as integrals over $\mathbb{R}^n \backslash B_\varepsilon(x)$ and $B_\varepsilon(x)$.

Make sure to review the class material available, in particular:

- Review the posted homework solutions.
- Review the posted class notes.
- Complement your study with the suggested practice problems.