VANDERBILT UNIVERSITY

MATH 3120 – INTRO DO PDES

Practice problems for HW 4

Question 1. Verify the properties of the inner product stated in class.

Question 2. Establish the following results stated in class.

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi x}{L}\right)\,dx = \begin{cases} 0, & m \neq n, \\ \frac{L}{2}, & m = n \neq 0, \\ L, & m = n = 0, \end{cases}$$

and

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)\,dx = \begin{cases} 0, & m \neq n, \\ \frac{L}{2}, & m = n. \end{cases}$$

Question 3. Show that

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)\,dx = 0,$$

where m and n are integers.

Question 4. Consider

$$f(x) = \begin{cases} x, & 0 \leq x \leq \frac{\pi}{2}, \\ \pi - x, & \frac{\pi}{2} < x \leq \pi. \end{cases}$$

Write f as a Fourier series in sin functions on the interval $[0, \pi]$, i.e.,

$$f(x) = \sum_{n=1}^{\infty} a_n \sin(nx).$$

Discuss the convergence of the series.

Question 5. Consider

$$f(x) = \cos^2(\pi x),\ 0 \leq x \leq 1.$$

Write f as a Fourier series in cos functions on the interval $[0, 1]$, i.e.,

$$f(x) = \sum_{n=0}^{\infty} a_n \cos(n\pi x).$$

Discuss the convergence of the series.

Question 6. Consider

$$f(x) = 1.$$

Write f as a Fourier series in sin functions on the interval $[0, L]$, i.e.,

$$1 = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right).$$

Discuss the convergence of the series.

Question 7. Consider

$$f(x) = 1.$$

Write f as a Fourier series in cos functions on the interval $[0, L]$, i.e.,

$$1 = \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right).$$

Discuss the convergence of the series. Compare your result with problem 6 and discuss the difference.

Question 8. Use separation of variables to solve the problem

$$\begin{aligned} u_t - u_{xx} &= 0 && \text{in } (0, L) \times (0, \infty), \\ u(x, 0) &= f(x) && 0 \le x \le L, \\ u_x(0, t) &= 0 && t \ge 0, \\ u_x(L, t) &= 0 && t \ge 0. \end{aligned}$$