VANDERBILT UNIVERSITY

MATH 3120 – INTRO DO PDES

HW 4

Question 1. Use separation of variables to solve the following initial-boundary value problem for the wave equation (the only difference from what was done in class is the boundary condition):

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= 0 && \text{in } (0,L)\times(0,\infty),\\ u(x,0) &= f(x) && 0 \le x \le L,\\ u_t(x,0) &= g(x) && 0 \le x \le L,\\ u_x(0,t) &= 0 && t \ge 0,\\ u_x(L,t) &= 0 && t \ge 0. \end{aligned}$$

Question 2. Show that the solution you found in problem 1 can be written as a superposition of a forward and a backward wave.

Question 3. Solve problem 1 with $c = 1$, $L = \pi$, $f(x) = \sin^3 x$, and $g(x) = \sin(2x)$.

Question 4. Use separation of variables to solve the following initial-boundary value problem for the heat equation:

$$\begin{aligned} u_t - k u_{xx} &= 0 && \text{in } (0,L)\times(0,\infty),\\ u(x,0) &= f(x) && 0 \le x \le L,\\ u(0,t) &= 0 && t \ge 0,\\ u(L,t) &= 0 && t \ge 0. \end{aligned}$$

Interpret your result.

Question 5. Solve problem 4 with $k = 17$, $L = \pi$, and

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{\pi}{2},\\ 2, & \frac{\pi}{2} < x \le \pi. \end{cases}$$

Discuss the convergence of the solution you found.