MAT 303 - Summer 10 — Practice Midterm

NAME:

Question 1. Classify the differential equations below as linear or non-linear and state their order.
(a) $y' + \cos(y) = 0$
(b) $\frac{d^2x}{dt^2} + \omega^2 x = 10\sin(t)$
(c) $yy'' + \frac{1}{x} = y$
(d) $e^{\sin x^2}\frac{dy}{dx} + xy = e^{-x}$

Question 2. The acceleration of an object moving in a straight line is proportional to the logarithm of the time elapsed since its departure. Find an equation for its position after time t. Is this a well defined problem?

Question 3. The Ancient footprints of Acahualinca are petrified Paleo-Indian human footprints in Managua, Nicaragua, left behind in volcanic ash and mud that solidified shortly after the group of up to 15 people passed by. The sand directly under the footprints contains 48% of the quantity of ^{14}C found in samples of the same materiel produced at present date. How old are the footprints (use $k = 0.0001216$)?

Question 4. Solve the following initial value problems.
(a) $xy' + (2x - 3)y = 5x^5$, $y(1) = 1$
(b) $(4xy + 6y^2)y' + 3x^2 + 2y^2 = 0$, $y(0) = 2$
(c) $y'' = 2yy'$, $y(-1) = 3$, $y'(-1) = 0$
(d) $xyy' = x^2 + y^2$, $y(1) = 1$

Question 5.. A 100ℓ tank initially contains $10kg$ of salt dissolved in 50ℓ of water. Brine containing $1kg/\ell$ of salt flows into the tank at the rate $2\ell/$ min, and the well-stirred mixture flows out of the tank at the rate $1\ell/$ min.

How much salt does the tank contain when 90% of its capacity is full?

Question 6. Consider a second order homogeneous linear differential equation. Show that any linear combination of two solutions is also a solution. Can you make a similar statement for higher order equations?

Question 7. Find the general solution of the following differential equations:
(a) $y'' + 16y = 0$
(b) $y'' + 4y' + y = 0$
(c) $y'' + 9y' = 0$
(d) $y'' - y' + 2y = 0$

Question 8. Find an example of a differential equation modeling some phenomena on your own field of study (e.g., if you are a business major you can find a differential equation describing compound interest). You do not have to derive the equation but rather understand what it is describing and why the equation takes that form. Try to avoid using examples already discussed in class.