VANDERBILT UNIVERSITY

MATH 2610 – ORDINARY DIFFERENTIAL EQUATIONS

Practice for test 2

The second test will cover all material discussed from (including) section 4.6 to (including) section 9.8, with the exception of the Cauchy-Euler equation (i.e., Cauchy-Euler will not be on the test), plus sections 1.3 and 1.4.

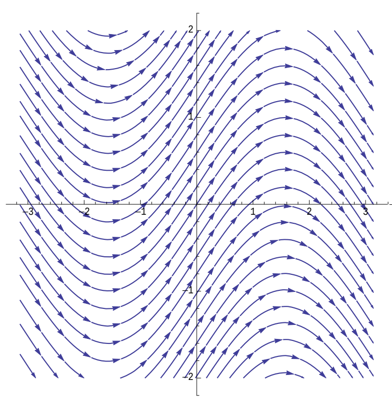
Question 1. Consider the equation

$$x^2 y'' - 2y = 0,\ x > 0.$$

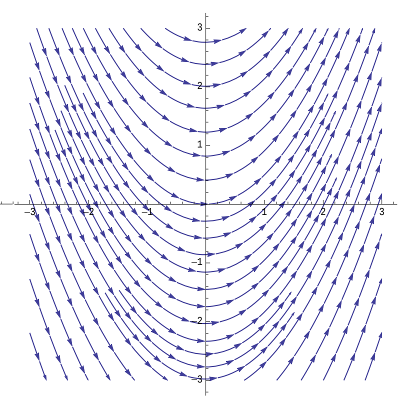
The functions $y_1 = x^2$ and $y_2 = x^{-1}$ are solutions of the differential equation (you do not have to show this). Are y_1 and y_2 linearly independent?

Question 2. Match the direction fields with the given differential equations.

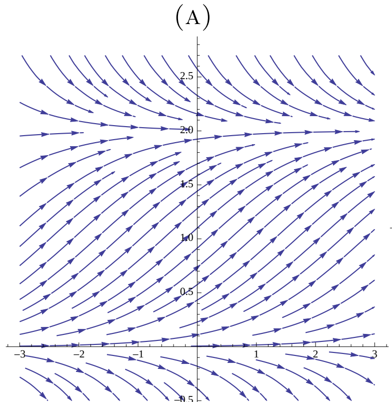
(a) $y' = -\dfrac{y}{x}$ (b) $y' = \cos x$ (c) $y' = y(1 - 0.5y)$ (d) $y' = x$



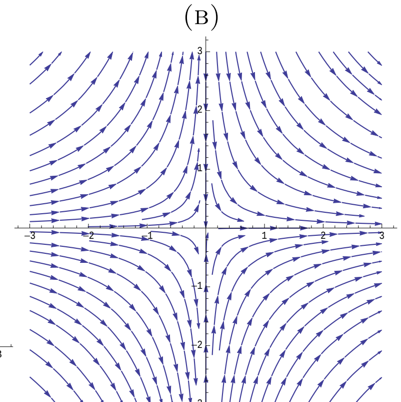
(A)



(B)



(C)



(D)

Question 3. For the systems $x' = Ax + f$ given below:

(a) Find the general solution if

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \text{ and } f(t) = e^{-2t} \begin{bmatrix} t \\ 3 \end{bmatrix}.$$

(b) State the form of the particular solution if

$$A = \begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix} \text{ and } f(t) = \begin{bmatrix} t^2 \\ t+1 \end{bmatrix}.$$

Question 4. Determine e^A if

$$A = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}.$$

Question 5. (a) Let $x_1, \cdots, x_k$ be vector functions defined on an interval I. State what it means for $x_1, \cdots, x_k$ to be linearly independent on I.

(b) Are $(e^t, -e^t)$ and $(5e^t, e^t)$ linearly independent on $(-\infty, \infty)$?

(c) Give an example of vector functions that are linearly independent on $(-\infty, \infty)$ but are linearly dependent on $(0, \infty)$.

Question 6. (a) Let $x_1, \cdots, x_k$ be vector functions defined on an interval I. State the definition of the Wronskian of $x_1, \cdots, x_k$.

(b) Prove that if the Wronskian of $x_1, \cdots, x_k$ does not vanish at a point $t_0 \in I$, then $x_1, \cdots, x_k$ are linearly independent on I.

Question 7. Let A be a $n \times n$ continuous matrix function.

(a) What is a fundamental matrix for the system $x' = Ax$?

(b) If X is a fundamental matrix for $x' = Ax$, show that $X' = AX$.

(c) Let X and Y be two fundamental matrices for $x' = Ax$. Show that there exists a constant matrix M such that $X = YM$.

Question 8. Let A be a $n \times n$ continuous matrix function and f be a continuous vector function, both defined on an interval I.

(a) State the variation of parameters formula for a particular solution to the system $x' = Ax + f$.

(b) Prove the formula you stated in (a).

Question 9. True or false?

(a) Every $n \times n$ matrix of real numbers has n linearly independent eigenvectors.

(b) If A is a $n \times n$ matrix of real numbers and x is a vector function that satisfies the initial value problem $x' = Ax$, $x(0) = 0$, then $x(t) = 0$ for all t.

(c) If A is a 3×3 matrix of real numbers whose eigenvalues are 2 with multiplicity two and 1, then A has two linearly independent generalized eigenvectors that correspond to the eigenvalue 2.

(d) The Wronskian of n linearly independent vector functions on an open interval I is never zero on I.

(e) If a 3×3 matrix of real numbers has eigenvalues $2+3i$, $2-3i$, and 5, then the matrix has three linearly independent eigenvectors.

Question 10. Know the theorems and definitions stated in class. Be prepared to state and use any of the theorems discussed in class, and to prove any theorem that has been proved in class or in an exercise. Also review the homework problems and posted examples.