VANDERBILT UNIVERSITY

MATH 2420 – METHODS OF ORDINARY DIFFERENTIAL EQUATIONS

Practice for test 1

Directions: This practice test should be used as a study guide, illustrating the concepts that will be emphasized in the test. This does not mean that the actual test will be restricted to the content of the practice. Try to identify, from the questions below, the concepts and methods that you should master for the test. For each question in the practice test, study the ideas and techniques connected to the problem, even if they are not directly used in your solution.

Take this also as an opportunity to practice how you will write your solutions in the test. For this, write clearly, legibly, and in a logical fashion. Make precise statements (for instance, write an equal sign if two expressions are equal; say that one expression is a consequence of another when this is the case, etc.).

The first test will cover all material discussed up to (including) section 4.7.

Question 1. For each equation below, identify the unknown function, classify the equation as linear or non-linear, and state its order.

(a) $\sqrt{y}\dfrac{dy}{dx} + x^2 y = 0.$

(b) $u'' + u = \cos x$

(c) $x''' = -\sin x \, x'.$

Question 2. Find the general solution of the given differential equation.

(a) $x' - x^2 = 0.$

(b) $x'' + 5x' + 6x = e^{2t}.$

(c) $y' = -\dfrac{4x^3 + y}{4y^3 + x}.$

Question 3. Give the form of the particular solution for the given differential equations. You do not have to find the values of the constants of the particular solution.

(a) $x'' - 3x' + 2x = e^{2t}.$

(b) $x'' + 9x = \sin t.$

(c) $x'' - x = 3t^2 + 1.$

(d) $x'' + x' - 2x = e^{-2t} + e^t$.

Question 4. Consider the following initial value problem:

$$y' - \sqrt{y} - xy^2 = 0,$$
$$y(1) = a.$$

Determine for which values of a this problem admits a unique solution.

Question 5. Find a particular solution to:

$$x'' + 9x = \sec^2(3t)$$

Question 6. Verify that $x_1(t) = t$ is a solution to

$$x'' - \frac{1}{t}x' + \frac{1}{t^2}x = 0,\ t > 0,$$

and find a second linearly independent solution.

Question 7. True or false? Justify your answers.

(a) If $p(x)$ and $q(x)$ are continuous functions on the interval (a, b), then the initial value problem

$$y'(x) + p(x)y(x) = q(x),$$
$$y(x_0) = y_0,$$

always admits a unique solution for any given $x_0 \in (a, b)$ and $y_0 \in \mathbb{R}$.

(b) Given the equation

$$M(x, y)\, dx + N(x, y)\, dy = 0,$$

it is always possible to find a function $F = F(x, y)$ such that $\frac{\partial F}{\partial x} = M$, $\frac{\partial F}{\partial y} = N$, and the general solution of the differential equation is given by $F(x, y) = C$, where C is an arbitrary constant.

(c) If a, b, and c are constants and $a \neq 0$, the equation

$$ax'' + bx' + cx = 0,$$

always admits two linearly independent solutions $x_1(t)$ and $x_2(t)$ that are defined for all $t \in \mathbb{R}$.

(d) If x_1 and x_2 are two functions such that their Wronskian vanishes, then they are linearly dependent.

Question 8. State and prove the superposition principle for second order linear differential equations with constant coefficients.

Question 9. Know the statements and how to use the theorems established in class. For theorems that were proved in class, know their proofs.

Question 10. Review the homework problems and examples posted in the course webpage.