VANDERBILT UNIVERSITY

MATH 2420 – METHODS OF ORDINARY DIFFERENTIAL EQUATIONS

Examples of sections 7.4 and 7.5

Question. Use the Laplace transform to solve

$$y'' + y = t^2 + 2,\ y(0) = 1,\ y'(0) = -1.$$

Solution. Taking the Laplace transform on both sides of the equation,

$$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{t^2 + 2\}.$$

Using the property

$$\mathcal{L}\{y''\} = s^2\mathcal{L}\{y\} - sy(0) - y'(0),$$

and $\mathcal{L}\{t^2\} = \frac{2}{s^3}$, $\mathcal{L}\{2\} = \frac{2}{s}$, we find

$$(s^2 + 1)Y(s) = s - 1 + \frac{2s^2 + 2}{s^3},$$

or

$$Y(s) = \frac{s}{s^2 + 1} - \frac{1}{s^2 + 1} + \frac{2}{s^3},$$

where $Y(s) = \mathcal{L}\{y\}$. Thus,

$$y(t) = \mathcal{L}^{-1}\{\frac{s}{s^2 + 1}\} - \mathcal{L}^{-1}\{\frac{1}{s^2 + 1}\} + \mathcal{L}^{-1}\{\frac{2}{s^3}\} = \cos t - \sin t + t^2.$$