## VANDERBILT UNIVERSITY

## MATH 2420 – METHODS OF ORDINARY DIFFERENTIAL EQUATIONS

Examples of section 7.2

Question. Suppose that the Laplace transform  $\mathcal{L}{f}(s) = F(s)$  exists for  $s > \alpha$ . Show that  $\mathcal{L}{e^{at}f(t)} = F(s-a),$ 

for  $s > \alpha + a$ .

Solution. We use the definition of the Laplace transform to compute

$$\mathcal{L}\{e^{at}f(t)\} = \int_0^\infty e^{-st} e^{at}f(t) dt$$
$$= \int_0^\infty e^{-(s-a)t}f(t) dt$$
$$= \int_0^\infty e^{-rt}f(t) dt,$$

where r = s - a. The last integral is the Laplace transform of f with independent variable r, i.e.,

$$\int_0^\infty e^{-rt} f(t) \, dt = F(r).$$

Since r = s - a, we conclude

$$\mathcal{L}\lbrace e^{at}f(t)\rbrace = \int_0^\infty e^{-rt}f(t)\,dt = F(r) = F(s-a).$$

F(r) is defined for  $r > \alpha$ , thus  $\mathcal{L}\{e^{at}f(t)\}$  is defined for  $s > \alpha + a$ .