VANDERBILT UNIVERSITY

MATH 2420 – METHODS OF ORDINARY DIFFERENTIAL EQUATIONS

Examples of section 4.4

Question 1. Write the form of the particular solution for the equations below (you do not have to find the values of the constants).
(a) $y'' + 9y = 2\cos(3x) + 3\sin(3x)$.
(b) $y'' + 9y = 2x^2 e^{3x} + 5$.

SOLUTIONS.

1a. The homogeneous equation is

$$y'' + 9y = 0,$$

with characteristic equation

$$\lambda^2 + 9 = 0,$$

whose roots are $\pm 3i$. Hence

$$y_1 = \cos(3x),\ \ y_2 = \sin(3x),$$

are solutions of the homogeneous equation. Given the form of $f(x)$, we look for

$$y_p = x^s\big(A\cos(3x) + B\sin(3x)\big).$$

Since $\cos(3x)$ and $\sin(3x)$ are solutions of the homogeneous equation, we need $s = 1$, so

$$y_p = x\big(A\cos(3x) + B\sin(3x)\big).$$

1b. The homogeneous equation is

$$y'' + 9y = 0,$$

with characteristic equation

$$\lambda^2 + 9 = 0,$$

whose roots are $\pm 3i$. Hence

$$y_1 = \cos(3x),\ \ y_2 = \sin(3x),$$

are solutions of the homogeneous equation. Given the form of $f(x)$, we look for

$$y_p = x^s A + x^r\big(Bx^2 + Cx + D\big)e^{3x}.$$

Since there is no repetition with the solutions of the homogeneous equation, $r = s = 0$ and

$$y_p = A + \big(Bx^2 + Cx + D\big)e^{3x}.$$