VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 15.9

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Question 1. Evaluate

$$\iint_R e^{x+y}\, dA,$$

where D is given by $|x| + |y| \leq 1$.

Solution 1. The region D is the square in the xy-plane with vertices $(1,0)$, $(0,1)$, $(-1,0)$, and $(0,-1)$. Let $u = x+y$ and $v = -x+y$. Then $u+v = 2y$, $u-v = 2x$, and therefore

$$x = \frac{1}{2}(u-v),$$

and

$$y = \frac{1}{2}(u+v).$$

We see that

$$|u| = |x+y| \leq |x| + |y| \leq 1 \Rightarrow -1 \leq u \leq 1,$$

and

$$|v| = |-x+y| \leq |x| + |y| \leq 1 \Rightarrow -1 \leq v \leq 1,$$

and we conclude that D is the image through the transformation $T(u,v) = (x,y)$ of the square with vertices $(1,1)$, $(1,-1)$, $(-1,-1)$, and $(-1,1)$ in the uv-plane.

Computing the Jacobian,

$$\frac{\partial(x,y)}{\partial(u,v)} = \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} = \det \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{2}.$$

Therefore

$$\iint_R e^{x+y}\, dA = \frac{1}{2} \int_{-1}^{1} \int_{-1}^{1} e^u \, du dv = e - e^{-1}.$$