VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 15.8

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Question 1. Using spherical coordinates, evaluate

$$\iiint_D \sqrt{x^2+y^2+z^2}\, dV,$$

where D lies above the cone $z = \sqrt{x^2+y^2}$ and between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$.

Solution 1. In spherical coordinates, $z = \sqrt{x^2+y^2}$ is equivalent to $\phi = \frac{\pi}{4}$, and $\sqrt{x^2+y^2+z^2} = \rho$. The region D is then given by

$$D = \{(\rho,\theta,\phi)\,|\,1 \leq \rho \leq 2, 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \frac{\pi}{4}\}.$$

Thus,

$$\begin{aligned}\iiint_D \sqrt{x^2+y^2+z^2}\, dV, &= \int_0^{\frac{\pi}{4}} \int_0^{2\pi} \int_1^2 \rho\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi \\ &= \int_0^{\frac{\pi}{4}} \sin\phi \, d\phi \int_0^{2\pi} d\theta \int_1^2 \rho^3 \, d\rho \\ &= \frac{15\pi}{2}(1-\frac{\sqrt{2}}{2}).\end{aligned}$$