

VANDERBILT UNIVERSITY, MATH 2300-04, F 20
EXAMPLES OF SECTION 15.2

MARCELO M. DISCONZI

Question 1. Evaluate

$$\iint_D x^2 dA,$$

where D is the region inside the square with vertices $(1, 1)$, $(-1, 1)$, $(-1, -1)$, and $(1, -1)$, and outside the square with vertices $(0, 1)$, $(-1, 0)$, $(0, -1)$, $(1, 0)$.

Solution 1. The region D can be written as $D = D_1 \cup D_2 \cup D_3 \cup D_4$, where

$$D_1 = \{(x, y) \mid 0 \leq x \leq 1, -x + 1 \leq y \leq 1\},$$

$$D_2 = \{(x, y) \mid -1 \leq x \leq 0, x + 1 \leq y \leq 1\},$$

$$D_3 = \{(x, y) \mid 0 \leq x \leq 1, -1 \leq y \leq x - 1\},$$

and

$$D_4 = \{(x, y) \mid -1 \leq x \leq 0, -1 \leq y \leq -x - 1\}.$$

Then

$$\begin{aligned} \iint_D x^2 dA &= \int_0^1 \int_{1-x}^1 x^2 dy dx + \int_{-1}^0 \int_{x+1}^1 x^2 dy dx \\ &\quad + \int_0^1 \int_{-1}^{x-1} x^2 dy dx + \int_{-1}^0 \int_{-1}^{-x-1} x^2 dy dx \\ &= 1. \end{aligned}$$